More on differentiation

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A problem from the other section

To determine the velocity of an eagle 10 seconds after it has left its nest, you set up a camera that takes two pictures within a short time interval Δt and activate the camera 10 seconds after the eagle leaves its nest. Using the two photos, you measure the distance Δy that the eagle has travelled between the two shots and you estimate its velocity by computing $\frac{\Delta y}{\Delta t}$.

From experience you know that the velocity of an eagle normally lies between $10\frac{m}{s}$ and $20\frac{m}{s}$.

In your first experiment you adjust the camera in such a way that the time interval Δt between the 2 shots is $1s$. In the second and third experiment you choose $\Delta t = 0.001s$ and $\Delta t = 100s$, respectively.

In which experiment do you expect to achieve the most accurate measurement for the velocity? Explain your answer.

Today we will learn how to differentiate logs and trigonometric functions, and also how to do implicit differentiation.

We'll start with the chain rule:

$$\frac{dy}{dx} = \frac{dy}{du}\,\frac{du}{dx}.$$

Here, y is a function of a function of x, something like

$$y = f(g(x)), \text{ i.e., } y = f(u) \text{ where } u = g(x).$$

Chain rule example

If $y = exp(x^2) = e^{x^2}$, we put $u = x^2$ and write $y = e^u$. Then $\frac{dy}{du} = e^u$, and also $\frac{du}{dx} = 2x$. Hence

$$\frac{dy}{dx} = e^u \cdot 2x = e^{x^2} 2x.$$

Another chain rule example

Suppose $y = \sqrt{1+\sqrt{x}}$. To differentiate y with respect to x, we write $u = 1+\sqrt{x}$ and say that $y = \sqrt{u}$. Then

$$\frac{dy}{du} = \frac{1}{2\sqrt{u}} = \frac{1}{2\sqrt{1+\sqrt{x}}},$$

while

$$\frac{du}{dx} = \frac{1}{2\sqrt{x}}.$$

It follows that

$$\frac{dy}{dx} = \frac{1}{2\sqrt{1+\sqrt{x}}} \cdot \frac{1}{2\sqrt{x}}.$$