Probability

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Math 10A
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We had a great breakfast this morning!

Pop-in lunch tomorrow at 12:30PM.

Pop-in lunch on Monday, November 7 at 12:10PM.

Breakfast next Thursday (Nov. 10) at 9:00AM.

Numberphile

Brady Haran from Numberphile may be popping in to class later today. He will probably be at lunch tomorrow, at least for the first half-hour.

Borrowed from Tuesday

If Ω is a probability space, a function

$$X : \Omega \to \mathbf{R}$$

is called a *random variable*.

A random variable is not random.

A random variable is not a variable.

A random variable is a function.

It is easy to give examples of random variabes in the Math 10B context. For example: $\Omega =$ the set of all 1024 outcomes of ten flips of a coin; $X(\omega) =$ the number of heads in ω.

It is arguably harder to find good examples of random variables in the “continuous” (Math 10A) situation.

Here's one: Let Ω be the set of places in the USA. (So Ω is the surface of that part of the earth that's include in the US.) For each $\omega \in \Omega$, let $X(\omega)$ be the temperature recorded at ω at midnight (GMT) on November 1. Note that X is recorded in Celsius and that $X(\omega)$ is measured at altitude of ω is on a mountain (say).

How is Ω a probability space? What is $P(A)$ if A is a subset of the US?

- We could let $P(A)$ be the area of A (normalized so that the total area of the USA is 1).
- We could let $P(A)$ be the population of A (normalized so that the total population of the US is 1).
- We could let $P(A)$ be the number of pieces of Halloween candy left in A at midnight on November 2, divided by the total number of pieces of Halloween candy in the US.

Let's stick with the first method.

What is $P(X \geq 20)$?

This is the fraction (measured in area) of the US where the temperature was at least 20°C at midnight GMT on November 1.

Note that the event $X \geq 20$ is the subset of the US where the temperature was 20 degrees C or above.

A PDF for X is a function $f(x)$ such that

$$P(X \in B) = \int_B f(x)\,dx$$

for subsets B of **R**. In particular, we should have

$$P(a \leq X \leq b) = \int_a^b f(x)\,dx$$

for real numbers a and b (with $a \leq b$).

There might be no PDF for a sufficiently annoying X.

A CDF for X is a function $F(x)$ such that $P(X \leq x) = F(x)$ for real numbers x. Then

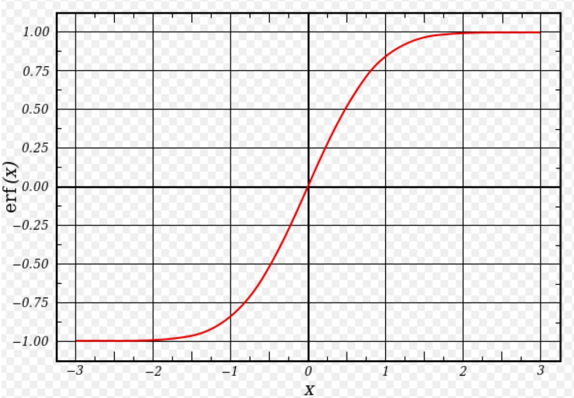
$$F(x) = \int_{-\infty}^x f(t)\,dt$$

is just an antiderivative of $f(x)$.

A CDF runs from 0 (for $x = -\infty$) to 1 (for $x = \infty$). For a slightly different normalization, see the Wikipedia article about the error function, which is defined by

$$\operatorname{erf}(x) = \frac{1}{\sqrt{\pi}} \int_{-x}^{x} e^{-t^2}\, dt$$

and has values running from -1 to $+1$ instead of from 0 to 1.



Pareto

Wikipedia: “The Pareto distribution, named after the Italian civil engineer, economist, and sociologist Vilfredo Pareto, is a power law probability distribution that is used in description of social, scientific, geophysical, actuarial, and many other types of observable phenomena.”

This distribution is given by its PDF, which is 0 for $x < 1$ and $\dfrac{p-1}{x^p}$ for $x \geq 1$. Here p is a parameter (as in the p-series test) that needs to be greater than 1. For example, if $p = 2$, the PDF is $\dfrac{1}{x^2}$ for $x \geq 1$.

MIT economics: “The right tail of income and wealth distributions often resemble Pareto."

Mean

For X with PDF equal to $f(x)$, the *mean* or average value of X is $\int_{-\infty}^{\infty} x \cdot f(x)\,dx$. The values of X are weighted according to how often they occur.

This is based explained in analogy with the "discrete" case, where the values of X are finite (or countably infinite).