Math 10A Fall, 2016

Professor Ken Ribet

Homework due Friday, October 7, 2016

> These problems are basically the problem set for Richard Bamler's Math 10A class. His students will be handing in their problems on October 4. Prof. Bamler encourages (or maybe even requires) his students to work on these problems *in groups.* As I have told quite a few of you in recent works, working with a group is an excellent way to master the material for this course. It's totally up to you whether or not you want to work with others. However, if you do work with a group, I suggest that you begin your homework by writing "I worked with X, Y and Z" (or whatever). Also—and this is really important to me—I strongly, strongly encourage you to write out answers to problems on your own, even if you have worked with a group. If you basically copy down what a classmate has written, you will be short-changing yourself.

The first part of the homework is to do these problems from §5.1 of the textbook: 1, 3, 5, 11, 15, 20, 24, 28, 34, 35, 36, 46, 49 These problems concern finding the *antiderivates* of various functions—the antiderivate of $f(x)$ is a function whose derivative is $f(x)$. We (I, I hope!) will be lecturing on this material next week, but I think it reasonable that you get a head start by trying to work these problems *now.*

The second part is to decide whether or not each of the following series converges:

1. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n \cdot 3^n}$

2. $\displaystyle\sum_{n=1}^{\infty} e^{\frac{1}{n}}$

3. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(\sqrt{2})^n}$

4. $\displaystyle\sum_{n=0}^{\infty} \frac{\pi^n}{3^{2n+1}}$. Hint: $3^{2n+1} = a \cdot b^n$ for some numbers a and b...

5. $\sum_{k=2}^{\infty} \frac{k^2}{k^2-1}$

6. $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1} \right)$. Hint: Compute the partial sums.

7. $\sum_{m=1}^{\infty} \ln \frac{m}{m+1}$ Hint: $\ln \frac{x}{y} = \ln x - \ln y$, if x and y are positive real numbers.

8. $\sum_{i=1}^{\infty} \left(\frac{5}{3^i} + \frac{2}{i} \right)$

The final part of the homework is to work out the following problems, which Prof. Bamler describes as "hard." For hard problems, it might be an especially good idea to consult with classmates, GSIs and me (yours truly, Ken R.).

9. Consider the following series

$$\frac{1}{1} + \frac{1}{2} + \frac{1}{2} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \dots$$

(so $\frac{1}{n}$ occurs n times for each n). Explain why this series does not converge.

10.

$$f : (-1, 1) \to \mathbf{R}, \qquad f(x) = \frac{1}{1 - x^3}.$$

Compute the tenth derivative of $f(x)$ at 0, $f^{(10)}(0)$. Here, $\mathbf{R}$ is the set of real numbers; there's a hint, which is to think of geometric series.

11. *(Very hard problem)* Use the figure below to explain why

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots$$

converges (Source: mathoverflow).