Mathematics 254a Exercises

September 27, 2007

1. Let k be a field, let $\overline{k}$ be an algebraic closure of k, and let A be a finite dimensional k–algebra. If $a \in A$, recall that $Nm_{A/k}(a)$ is by definition the determinant of the k-linear endomorphism $a_A \colon A \to A$ (multiplication by a). Let $S(\overline{k})$ be the set of k-homomorphisms from $A \to \overline{k}$ (the "geometric points" of $\operatorname{Spec} A$ in $\overline{k}$. Find and prove a formula for the image of $Nm_{A/k}(a)$ in $\overline{k}$ in terms of $S(\overline{k})$ and some "multiplicities" attached to each $\sigma \in S(\overline{k})$.

2. In the situation above, suppose that B/A is a finite and projective A-algebra. Can you prove that $Nm_{A/k} Nm_{B/A} = Nm_{B/k}$ using your formula? Can you prove this assuming that A/k or B/k is separable? Can you prove it without assuming that k is a field?

3. Neukirch, page 15: 1–3

4. Neukirch, page 23: 1, 2, 3, 4, 9, 10