Let $\mathcal{C}$ be a category with the property that for every pair of objects A and B, the class of morphisms from A to B is a set. Let $\hat{\mathcal{C}}$ be the category of *presheaves* on $\mathcal{C}$, that is, the category whose objects are the functors from $\mathcal{C}$ to the category $\mathcal{E}ns$ of sets and whose morphisms are natural transformations. For objects T and X of $\mathcal{C}$, let $h_X(T) := Mor_{\mathcal{C}}(T, X)$. If $f: T' \to T$ is a morphism in $\mathcal{C}$ and $g \in h_X(T)$, then $g \circ f \in h_X(T')$, so we have a map of sets $h_X(f): h_X(T') \to h_X(T)$. Check that these constructions define a contravariant functor h_X from $\mathcal{C}$ to $\mathcal{E}ns$, that is, a presheaf on $\mathcal{C}$. Check that a morphism $X \to X'$ defines a natural transformation $h_X \to h_{X'}$, so that h defines a functor $\mathcal{C} \to \hat{\mathcal{C}}$. Our aim is to prove that this functor is fully faithful.

Theorem: Suppose X is an object of $\mathcal{C}$ and F is an object of $\hat{\mathcal{C}}$.

1. If $\xi \in F(X)$, define a natural transformation $\eta(\xi): h_X \to F$ as follows. For every object T of $\mathcal{C}$, let

$$\eta(\xi)_T: h_X(T) \to F(T) : f \mapsto F(f)(\xi)$$

   (Check that this really does define a natural transformation!)

2. If $\eta: h_X \to F$ is a natural transformation, define $\xi(\eta) \in F(X)$ by

$$\xi(\eta) := \eta_X(\mathrm{id}_X)$$

Check that

1. For any ξ as in (1), $\xi(\eta(\xi)) = \xi$.
2. For any η as in (2), $\eta(\xi(\eta)) = \eta$.
3. The correspondences defined above are natural in a suitable sense.
4. When $F = h_{X'}$ for some object X' of X, the correspondence in (2) above

$$h_{X'}(X) = Mor_{\mathcal{C}}(X, X') \to \mathrm{Mor}_{\hat{\mathcal{C}}}(h_X, h_{X'})$$

   is the same as the correspondence defined by the functor h.

Conclusion: the functor $h: \mathcal{C} \to \hat{\mathcal{C}}$ is fully faithful.