Homework Assignment #10:

April 6, 2016

The first two of the following problems are taken from Eisenbud's book.

1. Let R be a noetherian local ring. Suppose that its maximal ideal is principal. Then every ideal is principal, and any nonzero ideal is a power of the maximal ideal. Conclude that the dimension of R is at most one.

2. Let k be a field. The following argument is an illustration of how Noether normalization can be used to compute the dimension of $k[x, y]$. Suppose that f is a nonzero element of $k[x, y]$. Show that for a suitable n, $k[x, y]/(f)$ is integral over the subring $k[x']$, where $x' := x - y^n$. Use this to prove that $k[x, y]$ has Krull dimension 2.

3. Let $A \to B$ be a flat and local homomorphism of noetherian local rings. Prove that $\dim B \geq \dim A$.

4. Let k be a field, let $A := k[x, y]/(y^2 - x^3 - x)$ and let m be the maximal ideal (x, y) of A. Show that $\mathrm{Gr}_m(A)$ is isomorphic to $k[t]$. Let $B := k[x, y]/(y^2 - x^3 - x^2)$. Show that $\mathrm{Gr}_m(B)$ is isomorphic to $k[s, t]/(st)$, assuming that the characteristic of k is not 2.