Math 1A — UCB, Spring 2010 — A. Ogus
Solutions[1] for Problem Set 9

§4.4 # 1(a). $\lim_{x\to a}\frac{f(x)}{g(x)}$=?
Solution. Because it is $\frac{0}{0}$ form, it is an indeterminate form. □

§4.4 # 1(b). $\lim_{x\to a}\frac{f(x)}{p(x)}$=?
Solution. The answer is 0 because $f(x)$ and $1/p(x)$ go to 0 when x goes to a. □

§4.4 # 1(c). $\lim_{x\to a}\frac{h(x)}{p(x)}$=?
Solution. Because it is $\frac{1}{\infty}$ form, the limit is 0. □

§4.4 # 1(d). $\lim_{x\to a}\frac{p(x)}{f(x)}$=?
Solution. Because it is $\frac{\infty}{0}$ form, the limit is ∞, $-\infty$ or does not exist. □

§4.4 # 1(e). $\lim_{x\to a}\frac{p(x)}{q(x)}$=?
Solution. Because it is $\frac{\infty}{\infty}$ form, it is an indeterminate form. □

§4.4 # 2(a). $\lim_{x\to a}[f(x)p(x)]$=?
Solution. Because it is $0\cdot\infty$, it is an indeterminate form. □

§4.4 # 2(b). $\lim_{x\to a}[h(x)p(x)]$=?
Solution. Because it is $1\cdot\infty$, the limit is ∞. □

§4.4 # 2(c). $\lim_{x\to a}[p(x)q(x)]$=?
Solution. Because it is $\infty\cdot\infty$, the limit is ∞. □

§4.4 # 9. $\lim_{x\to(\pi/2)^+}\frac{\cos x}{1-\sin x}$ =?
Solution. Because it is $\frac{0}{0}$ form, we can apply l'Hospital's rule.
$\lim_{x\to(\pi/2)^+}\frac{\cos x}{1-\sin x}=\lim_{x\to(\pi/2)^+}\frac{-\sin x}{-\cos x}=-\infty$. The last equality comes from the fact that $\cos x$ goes to 0 from the left when x goes to $\pi/2$ from the right(so, $1/\cos x$ goes to $-\infty$). □

§4.4 # 14. $\lim_{\theta\to(\pi/2)}\frac{1-\sin\theta}{\csc\theta}$ =?
Solution. The limit is 0 by the direct substitution property. □

§4.4 # 15. $\lim_{x\to\infty}\frac{\ln x}{\sqrt{x}}$ =?
Solution. Because it is $\frac{\infty}{\infty}$ form, we can apply l'Hospital's rule.
$\lim_{x\to\infty}\frac{\ln x}{\sqrt{x}}=\lim_{x\to\infty}\frac{1/x}{1/2\sqrt{x}}=\lim_{x\to\infty}\frac{2}{\sqrt{x}}=0$. □

§4.4 # 20. $\lim_{x\to 1}\frac{\ln x}{\sin\pi x}$ =?
Solution. Because it is $\frac{0}{0}$ form, we can apply l'Hospital's rule.
$\lim_{x\to 1}\frac{\ln x}{\sin\pi x}=\lim_{x\to 1}\frac{1/x}{-\pi\cos\pi x}=\frac{1}{\pi}$. □

§4.4 # 49. $\lim_{x\to\infty}(\sqrt{x^2+x}-x)$ =?
Solution. After factoring x out and changing the form as below,we can apply l'Hospital's rule because it is $\frac{0}{0}$ form.

$\lim_{x\to\infty}(\sqrt{x^2+x}-x) = \lim_{x\to\infty}\frac{\sqrt{1+1/x}-1}{1/x} = \lim_{t\to 0^+}\frac{\sqrt{1+t}-1}{t} = \lim_{t\to 0^+}\frac{1/2\sqrt{1+t}}{1} = \frac{1}{2}$. The second identity is just substitution $1/x$ by t. $\square$

§4.4 # 64. $\lim_{x\to\infty}\left(\frac{2x-3}{2x+5}\right)^{2x+1} = ?$

Solution. First, take the natural logarithm. Then, $\ln\left[\lim_{x\to\infty}\left(\frac{2x-3}{2x+5}\right)^{2x+1}\right] = \lim_{x\to\infty}\ln\left(\frac{2x-3}{2x+5}\right)^{2x+1} = \lim_{x\to\infty}\frac{\ln\left(\frac{2x-3}{2x+5}\right)}{1/2x+1}$. In the first equality, I use the fact that ln is continuous. Because the last form is $\frac{0}{0}$, we can apply l'Hospital's rule. Therefore, $\lim_{x\to\infty}\frac{\ln\left(\frac{2x-3}{2x+5}\right)}{1/2x+1} = \lim_{x\to\infty}\frac{\ln(2x-3)-\ln(2x+5)}{1/2x+1} = \lim_{x\to\infty}\frac{2/(2x-3)-2/(2x+5)}{-2/(2x+1)^2} = \lim_{x\to\infty}\frac{8(2x+1)^2}{(2x-3)(2x+5)}$. The last limit is 8.(You can easily figure it out by dividing the numerator and the denominator by x^2.) $\square$

§4.4 # 71. What happen if you try to use l'Hospital's Rule to evaluate

$$\lim_{x\to\infty}\frac{x}{\sqrt{x^2+1}}$$

. Evaluate the limit using another method.

Solution. If we derive the numerator and denominator, then we get $\frac{\sqrt{x^2+1}}{x}$. So, it does not help us. In this case, just divide the numerator and denominator by x. Then this limit goes to 1. $\square$

§4.4 # 72. If an object with mass m is dropped from rest, one model for its speed v after t seconds, taking air resistance into account, is

$$v = \frac{mg}{c}(1-e^{-ct/m})$$

where g is the acceleration due to gravity and c is a positive constant.
(a) Calculate $\lim_{t\to\infty} v$. What is the meaning of this limit?

Solution. $e^{-ct/m}$ goes to 0 when t goes to ∞. Thus, the limit is $\frac{mg}{c}$ and the meaning of this limit is the terminal speed of an object when it is dropped to the open space(no bottom). In this state, the air resistance is equal to the gravity. $\square$

(b) For fixed t, use l'Hospital's Rule to calculate $\lim_{c\to 0^+} v$. What can you conclude about the velocity of a falling object in a vacuum?

Solution. $\lim_{c\to 0^+} v = \lim_{c\to 0^+} mg\frac{1-e^{-ct/m}}{c} = mg\lim_{c\to 0^+}\frac{t/me^{-ct/m}}{1} = tg$. So, the velocity of a falling object in a vacuum is just tg. $\square$