Math 1A — UCB, Fall 2010 — A. Ogus
Solutions[1] for Problem Set 5

§2.8 # 1. Estimate the value of each derivative based on a graph
Solution.
(a) $f'(-3) = 1$
(b) $f'(-2) = \frac{2}{3}$
(c) $f'(-1) = 0$
(d) $f'(0) = -\frac{2}{3}$
(e) $f'(1) = 0$

□

§2.8 # 3. Match each graph with its derivative. Give reasons.
Solution. (a) goes with II, since (a) starts of decreasing (negative derivative), changes to increasing (positive derivative), and then back to decreasing, and II is the only graph that goes negative positive negative.

(b) goes with IV. (b) has two sharp corners, so it is not differentiable at those two points, and is straight in between. So the derivative should be three horizontal line segments with jump discontinuities in between.

(c) goes with I. Since (c) decreases on the left of the x-axis, and increases on the right, its derivative should be negative and then positive.

(d) goes with III. (d) has three flat spots (two local maximums and one minimum), so its derivative needs to cross the x-axis three times.

□

§2.8 # 5. Given the graph of f, sketch the graph of f'.
Solution. Since f increases very quickly on the right of the x-axis, has a maximum at $x = 0$, and then gradually decreases more and more slowly, the graph of f' should look almost flat and high for negative x, decrease very quickly to 0 and cross the origin, and then become a little negative and then increase slowly towards the x-axis (not crossing it).

□

§2.8 # 13. The graph shows the average age of some guys doing something. Sketch the graph of the derivative function $M'(t)$. During which years is the derivative negative?
Solution. The graph should be positive and flat-ish for most years, with a little bump dipping below the y-axis roughly between 1960 and 1970. Those are the years in which the derivative is negative.

□

§2.8 # 21. Find the derivative of $f(t) = 5t - 9t^2$ using the definition. State the domain of f and f'.
Solution.

$$
\begin{aligned}
f'(t) &= \lim_{h\to 0} \frac{(5(t+h)-9(t+h)^2)-(5t-9t^2)}{h} \\
&= \lim_{h\to 0} \frac{(5t+5h-9t^2-9th-9h^2)-(5t-9t^2)}{h} \\
&= \lim_{h\to 0} \frac{5h-9th-9h^2}{h} \\
&= \lim_{h\to 0} 5-9t-9h = 5-9t.
\end{aligned}
$$

The function and its derivative both have domain all real numbers.

□

§2.8 # 28. Find the derivative of $g(t) = \frac{1}{\sqrt{t}}$ using the definition. State the domain of g and g'.
Solution. clearing denominators and rationalizing, we find:

$$
\begin{aligned}
g'(t) &= \lim_{h\to 0} \frac{\frac{1}{\sqrt{t+h}} - \frac{1}{\sqrt{t}}}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{t}-\sqrt{t+h}}{h\sqrt{(t+h)t}} \\
&= \lim_{h\to 0} \frac{t-(t+h)}{h\sqrt{(t+h)t}(\sqrt{t}+\sqrt{t+h})} \\
&= \lim_{h\to 0} \frac{-h}{h\sqrt{(t+h)t}(\sqrt{t}+\sqrt{t+h})} \\
&= \lim_{h\to 0} \frac{-1}{\sqrt{(t+h)t}(\sqrt{t}+\sqrt{t+h})} \\
&= \frac{-1}{\sqrt{t^2}(\sqrt{t}+\sqrt{t})} = \frac{-1}{2t\sqrt{t}}.
\end{aligned}
$$

The function and its derivative both have domain all positive numbers, $(0, \infty)$.

□

§2.8 #41. Identify which curve is f, f' and f'' in the figure (in the book).
Solution.

$$
\begin{aligned}
a &= f \\
b &= f' \\
c &= f''
\end{aligned}
$$

c is decreasing close to $x = 0$. Therefore the derivative must be negative here, but both c and b are positive. That means that the derivative of c is not pictured, so $c = f''$. c is also negative at $x = 0$, so f' (whose derivative is c) is decreasing here. a is increasing at $x = 0$, so it must be that $b = f'$, which leaves $a = f$.

There are lots of other things to look at to figure this out, and to check that you're right.

□

§2.8 #44. The figure (in the book) shows four functions. One is position of some car, one is velocity, one is acceleration, and one is jerk. Figure out which is which.

Solution.

$d =$ position

$c =$ velocity

$b =$ acceleration

$a =$ jerk

a, b, and c all change from increasing to decreasing, and have a local maximum where the slope of the tangent line is zero. d is never zero, it does not cross the x-axis, and hence cannot be the derivative of any of the other functions. So d is position. d is strictly increasing, so its derivative must be entirely above the x-axis. Only c has this property, so c must be velocity. c has a maximum, where his derivative is zero, closer to the right side of the graph, where only b has a zero, and a is negative. That means b must be the derivative of c, acceleration, leaving a to be jerk.

$\square$