Math 1A — UCB, Spring 2010 — A. Ogus
Solutions[1] for Problem Set 13

§5.5 # 3. Evaluate the integral by making the given substitution.

$$\int x^2\sqrt{x^3+1}\,dx, \quad u = x^3+1$$

Solution. We have $u = x^3+1$, so $du = 3x^2\,dx$, or $du/3 = x^2\,dx$. Then

$$\int x^2\sqrt{x^3+1}\,dx = \int \sqrt{u}\frac{du}{3} = \frac{2}{9}u^{3/2} + C = \frac{2}{9}(x^3+1)^{3/2} + C$$

□

§5.5 # 5. Evaluate the integral by making the given substitution.

$$\int \cos^3(\theta)\sin\theta\,d\theta, \quad u = \cos\theta$$

Solution. We have $u = \cos\theta$, so $du = -\sin\theta\,d\theta$ or $-du = \sin\theta\,d\theta$. Then

$$\int \cos^3(\theta)\sin\theta\,d\theta = \int u^3(-du) = -\frac{1}{4}u^4 + C = -\frac{1}{4}\cos^4\theta + C$$

□

§5.5 # 11. Evaluate

$$\int (x+1)\sqrt{2x+x^2}\,dx$$

Solution. Try $u = 2x + x^2$. Then $du = (2+2x)\,dx = 2(1+x)\,dx$ or $du/2 = (x+1)\,dx$. So

$$\int (x+1)\sqrt{2x+x^2}\,dx = \int \sqrt{u}\,(du/2) = \frac{1}{3}u^{3/2} + C = \frac{1}{3}(2x+x^2)^{3/2} + C$$

□

§5.5 # 13. Evaluate

$$\int \frac{dx}{5-3x}$$

Solution. We can try $u = 5-3x$, so $du = -3\,dx$ or $-du/3 = dx$. Then

$$\int \frac{dx}{5-3x} = \int \frac{-du}{3u} = -\frac{1}{3}\ln u + C = -\frac{1}{3}\ln(5-3x) + C$$

□

§5.5 # 19. Evaluate

$$\int \frac{(\ln x)^2}{x}\,dx$$

Solution. We can use $u = \ln x$. Then $du = (1/x)\,dx$, so

$$\int \frac{(\ln x)^2}{x}\,dx = \int u^2\,du = \frac{1}{3}u^3 + C = \frac{1}{3}(\ln x)^3 + C$$

□

§5.5 # 29. Evaluate

$$\int e^{\tan x} \sec^2 x \, dx$$

Solution. We can use $u = \tan x$ so that $du = \sec^2 x \, dx$. Then

$$\int e^{\tan x} \sec^2 x \, dx = \int e^u \, du = e^u + C = e^{\tan x} + C$$

□

§5.5 # 43. Evaluate

$$\int \frac{1+x}{1+x^2} \, dx$$

Solution. Sometimes direct substitution isn't the best way to do a given problem. Instead, we can write

$$\int \frac{1+x}{1+x^2} \, dx = \int \frac{1}{1+x^2} \, dx + \int \frac{x}{1+x^2} \, dx$$

and then

$$\int \frac{1}{1+x^2} \, dx = \arctan x + C$$

and using $u = 1 + x^2$, we have $du = 2x \, dx$ or $du/2 = x \, dx$, so

$$\int \frac{x}{1+x^2} \, dx = \int \frac{du}{2u} = \frac{1}{2} \ln u + C = \frac{1}{2} \ln(1+x^2) + C$$

so

$$\int \frac{1+x}{1+x^2} \, dx = \arctan x + \frac{1}{2} \ln(1+x^2) + C$$

where we have chosen to combine the constants. □

§5.5 # 45. Evaluate

$$\int \frac{x}{(x+2)^{1/4}} \, dx$$

Solution. There aren't many options for substitution, but we could try $u = x + 2$. Then $du = dx$ and

$$\int \frac{x}{(x+2)^{1/4}} \, dx = \int \frac{u-2}{u^{1/4}} \, du = \int u^{3/4} - 2u^{-1/4} \, du = \frac{4}{7} u^{7/4} - \frac{8}{3} u^{3/4} + C = \frac{4}{7}(x+2)^{7/4} - \frac{8}{3}(x+2)^{3/4} + C$$

□

§5.5 # 64. Evaluate

$$\int_0^a x\sqrt{a^2 - x^2} \, dx$$

Solution. We should use $u = a^2 - x^2$, so $du = -2x \, dx$ or $-du/2 = x \, dx$. Then we plug the limits 0 and a into the equation $u = a^2 - x^2$ to get our new limits, a^2 and 0, respectively. (Keep them in the same order!)

$$\int_0^a x\sqrt{a^2 - x^2} \, dx = \int_{a^2}^0 \sqrt{u} \, \frac{-du}{2} = -\frac{1}{3} u^{3/2} \Big|_{a^2}^0 = -0 - -\frac{1}{3} a^3 = \frac{1}{3} a^3$$

□

§5.5 # 67. Evaluate

$$\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$$

Solution. This definitely calls for $u = \ln x$, so $du = dx/x$. Then we plug in the limits e and e^4 into $u = \ln x$ to get 1 and 4, respectively. Then

$$\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}} = \int_1^4 \frac{du}{\sqrt{u}} = 2u^{1/2}\Big|_1^4 = 2(2) - 2(1) = 2$$

□