There are 5 problems on this exam, with 12 parts in all. All problems have short solutions, except (5b). Work efficiently. If you don't know how to attack a problem, go on and come back to it later.

(1a) *7 points.* Use limit rules to evaluate $\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9}$. Show your steps.

(1b) *3 points.* Let $f(x) = \sqrt{x}$, with its natural domain. Does $f'(9)$ exist? Justify very briefly. *You may use problem (1a)!*

(2a) *6 points.* Let $f(x) = \frac{(x-2)(x-4)(x-8)}{3(x-1)(x-2)(x-8)}$, with its natural domain. Find all asymptotes of the graph of f. (You need not sketch the graph, just indicate the types and locations of the asymptotes.)

(2b) *6 points.* Find $\lim_{x\to 0} x^2 \sin(e^{1/x})$. Justify your answer briefly, using any limit rules or theorems from this course.

(3) *6 points.* Show that there is at least one real number x which satisfies $x^6 = 1 + \sin(x)$.

Short answer questions. Only very brief answers are required for these questions. You need not show your work or reasoning. *Each part is worth 2 points.*

(4a) Let $t > 0$. How is $\log_r(2)$ defined?

(4b) Let $f(x) = \tan(x)$ with domain $(-\pi, -\frac{\pi}{2})$. Does f have an inverse? If so, what are the domain and range of the inverse function?

(4c) If some *vertical* line intersects a graph at more than one point, what does this say about the graph?

(4d) Simplify: $\ln(5e\sqrt{x})$, assuming that $x > 0$.

(4e) If the domain of f contains $(-1, 1)$, and if f is continuous at 0, must $f'(0)$ exist? Either explain in words why it must exist, or give an example of a function for which it does not exist.

(5a) *5 points.* Let $f(x) = x^2$. Find $\delta > 0$ such that $|f(x) - 36| < \frac{1}{1000}$ whenever $|x - 6| < \delta$, and **show your reasoning** in full detail. *You need not simplify any numbers which arise, and there is no penalty if your δ is smaller than necessary.*

(5b) *7 points.* Show, using the precise definition of a limit, that

$$\lim_{x\to\frac{1}{3}} (9x - \frac{1}{x}) = 0.$$

(Here "show" means "prove".)