Quiz 1

This quiz is meant to test your clarity of writing as well as your understanding of the material. Write in complete sentences with correct grammar and punctuation.

1. Let a and b be integers. Define the *greatest common divisor* of a and b. Be sure to use the definition given in class.

   Definition: The *greatest common divisor* of two integers a and b is the unique natural number d satisfying the following two properties:

   - d divides a and b.
   - d is divisible by every integer d' which divides a and b.

   Note: This is the definition used in class. It is tempting, but not completely correct, to replace the second condition by saying that d is the greatest integer which divides a and b. This definition is not correct if a and b are both zero, and in any case it is not the one given in class. I gave half credit for this definition. Note that it is important to point out that d is uniquely characterized by these properties; for this to be true you have to specify that $d \geq 0$.

2. We know from a theorem in class that if S is the set of all linear combinations of a and b, then there exists a natural number d such that S is the set of all multiplies of d. Prove that this d is the greatest common divisor of a and b, in the sense of the definition you gave.

   Let d be such that S is the set of multiplies of d. Since a and b belong to S, d divides a and b. Now suppose that d' divides a and b. Since d belongs to S, there exist integers x and y such that $d = xa + yb$. This evidently implies that d' divides d.

   Note that this works even if $a = b = 0$. Here part of the point of this problem is to *prove* that the gcd of a and b exists. Some people assumed this existence, and then tried to prove that if c is the gcd, then S is the set of all multiplies of c. Others tried to prove that since S is closed under subtraction, there exists some c such that $S = c\mathbf{Z}$, which you were allowed to assume.

   The median score on this quiz was 10, the average 10.77. Don't worry, you can improve is you try! It's not so easy to learn to write mathematics precisely.