Anaximander's Saying

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εξ ων δε η γενεσις εστι τοις ουσι

και την φθοραν εις ταυτα γινεσθαι

κατα το χρεων

διδοναι γαρ αυτα δικην

και τισιν αλληλοις της αδικιας

κατα την του χρονου ταξιν

A translation:

Where the source of things is,

to that place they must also pass away,

according to necessity,

for they give justice

and make reparation to one another for their injustice,

according to the arrangement of Time.

This talk will give a reading of this saying of Anaximander using the language of group actions.

This reading would like to suggest that the Pre–Socratic Greek philosophers did encounter something quite definite that was clearly hard to express with the words available to them – something that is still hard to express but which can hopefully be more clearly indicated with the language we have now.

The Greeks in their day were immersed in this encounter; while nowadays just indicating this encounter falls far short of actually engaging in it.

Why group actions?

There are indications

from the geometry known
to the first Pre-Socratic Thales,

and from an analogy used by Parmenides,

that the Pre-Socratics were conscious of what we would call the rigid motions in 2 and 3 dimensions,

and were conscious of being immersed in something similar to them.

Thales (≈ 625 – 545 BC)

Anaximander (≈ 610 – 540 BC)

Parmenides (≈≈ 540 – 450 BC)

Geometry known to Thales
(according to a student of Aristotle):

that a circle is bisected by its diameter;

that in a isosceles triangle the angles at the base are equal;

that when two straight lines intersect the angles at the vertex are equal.

(Scholars have been reluctant to credit these results as proved by Thales in the Euclidean sense)

But this knowledge does suggest an ability to see the effects of the rigid motions of the plane.

Parmenides wrote one work in two parts –
The Way Of Truth and The Way Of Opinion.

Near the end of The Way Of Truth he uses a geometric analogy, comparing what's been discussed to

'the bulk of a well–rounded sphere,
everywhere from the center equally matched'

The word he uses to describe this is $\pi\epsilon\iota\rho\alpha\varsigma$.
This means variously:

bounded, limited, experienced.

It is also a word used in one of Anaximander's main sayings:

'The first principle of the things that exist is the $\alpha\pi\epsilon\iota\rho o\nu$'

(The α here acts as negation or privation)

If we generalize from rigid motions of the plane
(Thales' geometric discoveries)

or rigid motions of space
(Parmenides' simile)

to group actions,

then Parmenides' $\pi\epsilon\iota\rho\alpha\varsigma$ would refer to an orbit under a group action (the action being actually present as the stabilizer of the center). The orbit is then called limited, bounded, experienced.

Anaximander's first principle of the things that exist $\alpha\pi\epsilon\iota\rho o\nu$ would then refer to the lack of such an orbit and group action. The lack is then called unlimited, unbounded, unexperienced.

Some Group Action Preliminaries

Let G be a group acting on a set B.

G also acts on other sets (pairs from B, subsets, sets of pairs, functions, etc.)

G acts on $V[G]$ (the cumulative hierarchy over B)

For c in $V[B]$, the G–orbit of c is $G{\cdot}c = \{gc\,;\ g \in G\}$

For $c \in V[B]$, the stabilizer of c is
$G_c = \{g \in G\,;\ gc = c\}$

For $g \in G$ acting on a set C we let $g|C$ be the function $\{< c, gc >; c \in C\}$; and let $G|C = \{g|C; g \in G\}$

Notice for any G and set C we can form $G_C|C$. This is always a subgroup of the permutation group $Per(C)$.

For G acting on B, and $b, c \in V[B]$, there are three orbits associated with the action of G on b, c:

$Q_c = G_c \cdot b$ here G_c acts on Q_c

$R = G \cdot < b, c >$ here G acts on R

$S_b = G_b \cdot c$ here G_b acts on S_b

(There is an obvious symmetry between b and c here and hence between Q_c and S_b, but we will soon break this symmetry by construing b as the ground for c)

In model theory (under the action of the group G of automorphisms of a saturated model) these correspond to

Q	the type of b over c
R	the type of b, c
S	the type of c over a

For G a group acting on a set B,

i in $V[B]$ is G-invariant iff $G_i = G$.

The G-invariant objects remain fixed under the actions of G and may be thought of as providing the language appropriate for G:

For c in $V[B]$, the orbit $T = G{\cdot}c$ is G-invariant. Under the action of G, c varies through T, while T remains fixed.

For c in $V[B]$, the G_c-invariant objects are just G-invariant objects with the parameter c:

i is G_c-invariant iff
there is a G-invariant function f such that $f(c) = i$
(f is the G orbit $G\cdot < c, i >$)

So the language of G (ie G-invariance) can be viewed as providing a common basis throughout all the c in a G-orbit T for the various G_c-invariancies.

Notice by the way that
for any b,
and for a given value D for the stabilizer of b,

for any G such that $G_b = D$, we have that
G is D-invariant,
so G belongs to the language of D.

In particular for any H such that H_b also equals D,
there is a H invariant function f such that $f(b) = G$.

In this sense every such G is available through the
language of any such H.

Let's give an example of the intended meaning of the $\pi\epsilon\iota\rho\alpha\varsigma$ $\alpha\pi\epsilon\iota\rho o\nu$ theme in our proposed reading:

Consider a family $< D_b \mid b \in B >$ such that for some G a subgroup of $Per(B)$, for all $b \in B, \quad G_b = D_b$.

There are many possible G:

there is the minimal $G =$
the group generated by all the D_b;

there is the maximum $G =$
the automorphism group of $< D_b \mid b \in B >$;

and there is any group in between.

If the object of interest is the family of $\pi\epsilon\iota\rho\alpha\varsigma$
$< D_b \mid b \in B >$,
then any of these $\alpha\pi\epsilon\iota\rho o\nu$ group actions with their interplay of distinctions and lack of distinctions is allowable.

This situation arises in Model Theory:

for $\mathcal{M}$ a saturated model, let $B =$ the set of small size elementary submodels of $\mathcal{M}$.

Notice that $\mathcal{M}$ modulo algebraic equivalence is in $V[B]$.

Let $G =$ the group action on B induced by the automorphism group of $\mathcal{M}$.

In $< G_b \mid b \in B >$, the minimal group (generated by the G_b) gives rise to orbits of tuples from $\mathcal{M}$ that are the Lascar types.

The group G of automorphisms gives rise to orbits that are the usual first-order types for the model $\mathcal{M}$.

The maximal group $Aut(< G_b \mid b \in B >)$ ignores some of the distinctions made by G.

To generalize, let's consider:

a set A, and for each a in A an element b_a of B,

and a family $< D_a \mid a \in A >$ such that for some G a subgroup of $Per(B)$, $\quad G_{b_a} = D_a$ for all $a \in A$.

(So instead of having all the G_b, we only have some)

Any such group G can act on this:

for each a in A, G can send b_a to any β_a in the same G orbit, giving rise to a new family $< G_{\beta_a} \mid a \in A >$.

For this new family, there is a new collection of possible G.

We are now considering the situation of having a set A of (for want of a better word) entities;

at any moment they are at places in the set B and subject to the action of some group G.

Each a in A may encounter various $c \in V[B]$ (which means roughly that the stabilizer G_c must act on a in a specified way).

The current place $b \in B$ of each a with its corresponding stabilizer D plus the encounters of a, will (together with possibly other considerations) restrict the possible actions G.

(No attempt will be made to make this more precise: at any moment there will be a set of possible group actions available to a, and all of these are D invariant and thus part of a's language; so we will leave it up to the a's).

Our reading of Anaximander will be based on the point of view that there is no definite G acting on B (there are many possibilities for G) but in some sense (just indicated) for some b's in B G_b is known.

For c in $V[G]$, c is considered indefinite in itself (since for example its orbit depends on which G is used) but c grounded by such a b is definite to the extent that the G_b-orbit is known. This is part of the encounter with c at b.

The other part of an encounter is the G_c orbit of b.

Each possible G realizes the encounter through the G orbit of b, c.

(These are the S,Q and R referred to earlier).

The Anaximander fragment involves certain key terms: $\gamma\epsilon\nu\epsilon\sigma\iota\varsigma$, $\varphi\vartheta o\rho\alpha\nu$, and $\tau o\iota\varsigma\ o\upsilon\sigma\iota$

$\gamma\epsilon\nu\epsilon\sigma\iota\varsigma$ means genesis

$\varphi\vartheta o\rho\alpha\nu$ means destruction

$\tau o\iota\varsigma\ o\upsilon\sigma\iota$ means the things that are

To carry out the desired reading we will say now what these terms are to be interpreted as in the language of group actions.

We will interprete them roughly as follows:

$(\gamma\epsilon\nu\epsilon\sigma\iota\varsigma)$ as Q
$(\varphi\vartheta o\rho\alpha\nu)$ as S
and
$(\tau o\iota\varsigma\ o\upsilon\sigma\iota)$ as R

We will fix a set B
and consider subgroups G of $Per(B)$.

We wish to give a reading to the phrases
genesis and destruction as applied to the things that are
$(\gamma\epsilon\nu\epsilon\sigma\iota\varsigma, \varphi\vartheta o\rho\alpha\nu, \tau o\iota\varsigma\ o\upsilon\sigma\iota)$.

First: an element of $V[B]$ is not automatically a thing that is. We will take the elements b in B as providing the ground for the things that are.

So the things that are consist of c in $V[B]$ as grounded by a b in B.

The $\gamma\epsilon\nu\epsilon\sigma\iota\varsigma$ of b, c is the action $G_c|G_c{\cdot}b$

The $\varphi\vartheta o\rho\alpha\nu$ of b, c is the action $G_b|G_b{\cdot}c$

To elaborate:

Interpretation of $\gamma\epsilon\nu\epsilon\sigma\iota\varsigma$:

A thing that is consist of a c which has an impact on a b.

The impact here is: b is in an orbit of a particular group subaction of G, namely the action of G_c; in a sense c acts on b.

To b, though, this simply consist of b being confined to a particular set $Q = G_c \cdot b$ with a particular group $\Gamma = G_c|Q$ acting on Q.

So: b is in a set Q acted on by a Γ where Γ is a subgroup of $G_Q|Q$.

This Γ acting on Q is what we'll call the $\gamma\epsilon\nu\epsilon\sigma\iota\varsigma$ of the $\tau o\iota\varsigma\ o\upsilon\sigma\iota$.

(Notice there is no connection between c and Γ, Q apart from the one provided by G; c is not yet completely there at the $\gamma\epsilon\nu\epsilon\sigma\iota\varsigma$)

Interpretation of $\varphi\vartheta o\rho\alpha\nu$:

A thing that is has a presence for b even when that thing is in a sense completely absent except for the ground b itself.

We will call that presence
the $\varphi\vartheta o\rho\alpha\nu$ of this $\tau o\iota\varsigma\ o\upsilon\sigma\iota$
and define it to be
the action on $S = G_b{\cdot}c$ of the group $\Phi = G_b|S$.

Even though c is gone, its G_b orbit is G_b invariant; and so it is part of the language of G_b and so available to b.

The action of G on b, c can be analyzed in two ways:

if we first act on c but keep this action hidden, then all that is left of the action of G is the action of G_c on $G_c{\cdot}b$, ie Γ on Q, the $\gamma\epsilon\nu\epsilon\sigma\iota\varsigma$;

if we first act on b but keep this action hidden, then all that is left of the action of G is the action of G_b on $G_b{\cdot}c$, ie Φ on S, the $\varphi\vartheta o\rho\alpha\nu$.

in this way the actions Γ and Φ belong together as part of the common action G.

If G sends b, c to β, γ, then the Γ, Q of c still belongs together with the Φ, S at β.

We can now give our reading of the Anaximander saying:

The saying of Anaximander is in two sentences.

The sentences are connected by the word $\gamma\alpha\rho$, which indicates that the second sentence clarifies the first. $\gamma\alpha\rho$ can be translated as 'namely'.

Each sentence is in two parts connected by the word $\kappa\alpha\tau\alpha$, which means 'downward from' and can be translated as 'according to'.

So the saying has the form:

	Clause 1	$\kappa\alpha\tau\alpha$	Clause 2
$\gamma\alpha\rho$			
	Clause 3	$\kappa\alpha\tau\alpha$	Clause 4

Clause 1

$\epsilon\xi$ $\omega\nu$ $\delta\epsilon$ η $\underline{\gamma\epsilon\nu\epsilon\sigma\iota\varsigma}$
$\epsilon\sigma\tau\iota$ $\underline{\tau o\iota\varsigma\ o\upsilon\sigma\iota}$
$\kappa\alpha\iota$ $\tau\eta\nu$ $\underline{\varphi\vartheta o\rho\alpha\nu}$

$\epsilon\iota\varsigma$ $\underline{\tau\alpha\upsilon\tau\alpha}$ $\underline{\gamma\iota\nu\epsilon\sigma\vartheta\alpha\iota}$

The $\gamma\epsilon\nu\epsilon\sigma\iota\varsigma$ and $\varphi\vartheta o\rho\alpha\nu$ of $\tau o\iota\varsigma\ o\upsilon\sigma\iota$

are $\gamma\iota\nu\epsilon\sigma\vartheta\alpha\iota$ $\tau\alpha\upsilon\tau\alpha$ (as $\tau\alpha\upsilon\tau\alpha$)

The $\gamma\epsilon\nu\epsilon\sigma\iota\varsigma$ and $\varphi\vartheta o\rho\alpha\nu$ of $\tau o\iota\varsigma\ o\upsilon\sigma\iota$

are genesised there (as the same).

and this is $\kappa\alpha\tau\alpha$ (downward from, or, according to)

Clause 2

$\tau o\ \chi\rho\epsilon\omega\nu$

$\chi\rho\epsilon\omega\nu$ means: that which must be.

Here, given
a β with its Φ,S
and c with its Γ,Q
and given that these belong together,
that means there must be a G that joins them.

Any such G is that which must be, is $\chi\rho\epsilon\omega\nu$.

$\chi\rho\epsilon\omega\nu$ is usually translated as necessity; but the necessity here is that Γ,Q and Φ,S belong together, i.e. an encounter (which occurs without specifying a specific G).

Clause 3 involves the key terms $\delta\iota\kappa\eta$ and $\tau\iota\sigma\iota\nu$

$\delta\iota\kappa\eta$ has two main meanings:

usage, and, judgement

We will take it to refer to the group G, the particular choice of group.

For example: in model theory when studying a saturated $\mathcal{M}$, we take the group to be the automorphism group of $\mathcal{M}$.

But when we need the extra distinctions of Lascar types, we implicitly switch to that minimal group referred to earlier.

Whichever group we are dealing with at a given moment is $\delta\iota\kappa\eta$.

$\tau\iota\sigma\iota\nu$ means payment by way of return.

We will take it to refer to the belonging together of Γ and Φ from the first clause.

This joining is $\kappa\alpha\tau\alpha$, downward from, G.

For two groups G and H, each will bind things together in its own way. We will say that H gives payment by way of return to G when H allows a particular Γ and Φ to be joined downward from G rather than separated downward from H.

Clause 3

διδοναι γαρ αυτα <u>*δικην*</u>

και <u>*τισιν*</u> *αλληλοις της* <u>*αδικιας*</u>

αυτα giving *δικην*
and *τισιν* to each other for *αδικιας*

themselves giving *δικην*
and *τισιν* to each other for *αδικιας*

and this is $\kappa\alpha\tau\alpha$ (downward from, or, according to)

Clause 4

$\tau\eta\nu\ \tau o\upsilon\ \chi\rho o\nu o\upsilon\ \tau\alpha\xi\iota\nu$

$\chi\rho o\nu o\upsilon$ refers to time and temporality;

$\tau\alpha\xi\iota\nu$ refers to arrangement or order; it has certain military connotations too.

Together, we'll read these as referring to temporal strategies:

i.e., which *G* to be used when:

the *G*'s join together the $\gamma\epsilon\nu\epsilon\sigma\iota\varsigma$ and the $\varphi\vartheta o\rho\alpha\nu$ when they belong together.

the $\gamma\epsilon\nu\epsilon\sigma\iota\varsigma$ and the $\varphi\vartheta o\rho\alpha\nu$ are $\pi\epsilon\iota\rho\alpha\varsigma$, are matters of experience, as is their being joined, but the $\chi\rho\epsilon\omega\nu$, the *G* that joins them, is $\delta\iota\kappa\eta\nu$, is $\alpha\pi\epsilon\iota\rho o\nu$, not a matter of experience. And which and when they occur as $\delta\iota\kappa\eta\nu$ is a matter for $\chi\rho o\nu o\upsilon\ \tau\alpha\xi\iota\nu$.