Errata for "Nonarchimedean geometry of Witt vectors" (updated 5 Dec 2015)

In the proof of Theorem 5.11, the derivation of (5.11.1) is incorrect (as reported by Lizao Ye). To be precise, the inequality

$$\lambda(\alpha^s)(x_j y_k \pi^{j+k}) > \max_i \{\lambda(\alpha^s)(z_i \pi^i)\}$$

appearing near the bottom of the first paragraph does not follow from the previous arguments: it would follow if one had $\lambda(\alpha^s)(\pi) = p^{-1}$, but this fails for s small unless $\overline{\pi} = 0$.

To remedy this, we first verify (5.11.1) in the case where $x = x_i \pi^i$ for x_i stable and $y = 1$. Let $z_0, z_1, \dots$ be a presentation of $xy = x$ satisfying (5.11.2). If $i > 0$, then z_0 is divisible by π and $\lambda(\alpha)(z_0) = p^{-1}\lambda(\alpha)(z_0/\pi)$, so $0, z_0/\pi + z_1, z_2, \dots$ is another presentation satisfying (5.11.2). By repeating this argument, we see that the sequence $z'_0, z'_1, \dots$ given by

$$z'_j = \begin{cases} 0 & j < i \\ \pi^{-i}(z_0 + z_1\pi + \cdots + z_i \pi^i) & j = i \\ z_j & j > i \end{cases}$$

is also a presentation of x satisfying (5.11.2). Since $\lambda(\alpha)(x_i) > \lambda(\alpha)(z'_i)$, $x_i - z'_i$ is nonzero, divisible by π, and again stable (although not necessarily of the form given by Lemma 5.5). We may now obtain a contradiction either by considering Newton polygons (see Lemma 6.3), or by an explicit calculation as follows. Write

$$a = x_i - z'_i = \sum_{n=0}^{\infty} p^n [\overline{a}_n], \qquad b = a/\pi = \sum_{n=0}^{\infty} p^n [\overline{b}_n],$$

and take n to be the largest integer such that $p^{-n}\alpha(\overline{b}_n) = \lambda(\alpha)(b)$. Since $\overline{\pi}_1$ is a unit, we see that $\overline{a}_{n+1}$ is dominated by $\overline{b}_n \overline{\pi}_1$, so

$$\lambda(\alpha)(a) = p^{-1}\lambda(\alpha)(b) = p^{-n-1}\alpha(\overline{a}_{n+1}).$$

This contradicts the stability of a.

To now verify (5.11.1) in the general case, it suffices to obtain a contradiction under the assumption that (5.11.2) holds for some $t \in S$ (for S defined as in the original argument). By the previous paragraph,

$$H(\alpha, \pi, t)(x_j y_k \pi^{j+k}) = (t/p)^{j+k} \lambda(\alpha)(x_j y_k);$$

consequently, we have

$$\begin{aligned} H(\alpha, \pi, t)(x_j y_k \pi^{j+k}) &> \max_i \{H(\alpha, \pi, t)(z_i \pi^i)\}, \\ H(\alpha, \pi, t)(x_j y_k \pi^{j+k}) &> \max_{(j',k') \neq (j,k)} \{H(\alpha, \pi, t)(x_{j'} y_{k'} \pi^{j'+k'})\}. \end{aligned}$$

This gives a contradiction against the equality

$$x_j y_k \pi^{j+k} = \sum_{i=0}^{\infty} z_i \pi^i - \sum_{(j',k') \neq (j,k)} x_{j'} y_{k'} \pi^{j'+k'},$$

and (5.11.1) follows.

Additional corrections:

- Lemma 5.5: in the last sentence of the proof, the convergence is with respect to the componentwise topology, not the $(p, [\overline{\pi}])$-adic topology.