Clay Mathematics Institute

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2005

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Euclid: www.claymath.org/euclid
James Arthur Collected Works: www.claymath.org/cw/arthur
Hanoi Institute of Mathematics: www.math.ac.vn
Ramanujan Society: www.ramanujanmathsociety.org

James Carlson

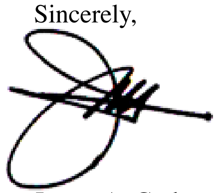
Dear Friends of Mathematics

This past year marked the sixth anniversary of the Clay Mathematics Institute's program of summer schools. The first, held at Pine Manor College, was dedicated to mirror symmetry. The last, held at MSRI in Berkeley, was on Ricci flow, 3-manifolds, and geometry. It was of special interest because of Perelman's proposed solution to the Poincaré conjecture and to Thurston's geometrization conjecture. It was also the culmination of Hamilton's Ricci flow program and the deep work of many mathematicians who have dedicated their life's work to geometry and analysis. The aim of the summer schools is to provide participants – graduate students and recent PhD's within five years of their degree – with the background needed to work successfully in an active and important area of mathematics. The next two schools, one on arithmetic geometry, the other on homogenous flows, dynamics, and number theory, will be held in Göttingen and Pisa, respectively.

In addition to major, ongoing activities such as the summer schools, the Institute undertakes a number of smaller special projects each year. Four examples demonstrate the breadth of the Institute's work. First is the digitization of the oldest surviving manuscript of Euclid's *Elements*, dated to 888 AD, when it was copied from a predecessor in Constantinople. For the first time, a facsimile of the founding document of mathematics is widely available. Second is the James Arthur Archive on the CMI website. This is an online library of the collected works of Toronto mathematician James Arthur, noted for his achievements in automorphic forms and representation theory. James Arthur will be adding further material to the archive, including his own comments. The Clay Mathematics Institute hopes that there will be further efforts of this kind, both at CMI and elsewhere. The third example is the Institute's support of George Csicsery's documentary film on the life and work of Julia Robinson, noted for her key role in the solution of Hilbert's tenth problem. Robinson's inspiring story will be of interest to both mathematicians and the general public. Finally, the Institute is supporting two small ventures in Vietnam and India. The first, part of Bao-Châu Ngô's research award in 2004, is a donation of books to the Mathematical Institute in Hanoi. The second, part of Manjul Bhargava's research award in 2005, is two years of support for the Ramanujan Mathematical Society (RMS) in Mumbai. The RMS will offer one or two Clay–Bhargava fellowships of 50,000 Rupees to promising graduate students.

The programs and projects mentioned above illustrate a guiding principle of the Clay Mathematics Institute: support a broad spectrum of initiatives in mathematics, large and small, in order to foster the creation of new knowledge.

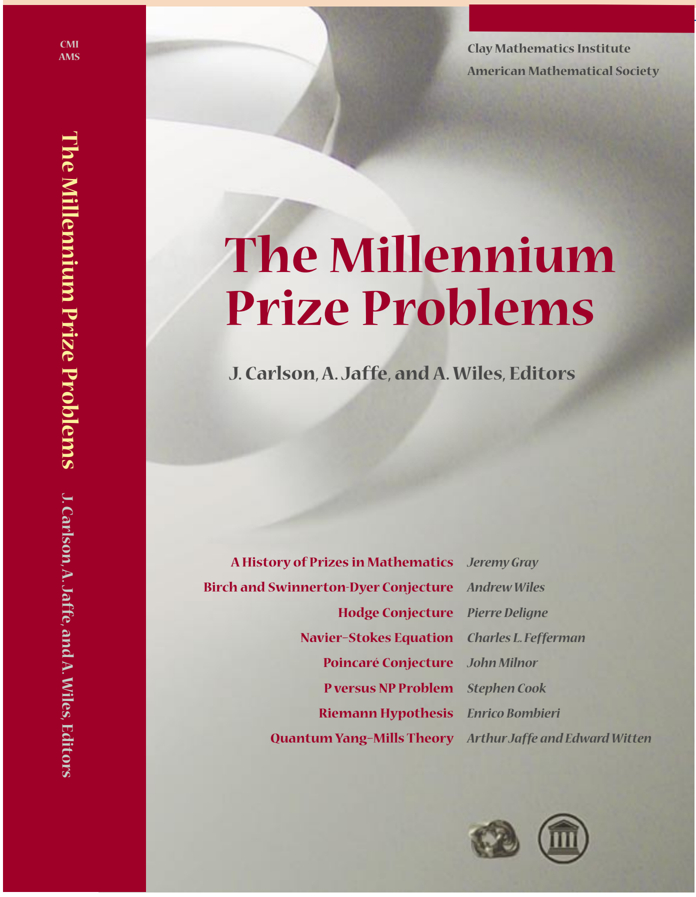
Sincerely,



James A. Carlson

The expected publication date is July 23, 2006. Copies of the book will be available at the ICM in Madrid, August 22–30, 2006, and also sold at www.ams.org/bookstore

The Millennium Prize Problems



On August 8, 1900, at the second International Congress of Mathematicians in Paris, David Hilbert delivered the famous lecture in which he described twenty-three problems that were to play an influential role in future mathematical research. A century later, on May 24, 2000, at a meeting at the Collège de France, the Clay Mathematics Institute announced the creation of a US$7 million prize fund for the solution of seven important classic problems that have resisted solution. The prize fund is divided equally among the seven problems. There is no time limit for their solution.

The Millennium Prize Problems were selected by the founding Scientific Advisory Board of CMI – Alain Connes, Arthur Jaffe, Andrew Wiles, and Edward Witten – after consultation with other leading mathematicians. Their aim was somewhat different than that of Hilbert: not to define new challenges, but to record some of the most difficult issues with which mathematicians were struggling at the turn of the second millennium; to recognize achievement in mathematics of historical dimension; to elevate in the consciousness of the general public the fact that in mathematics the frontier is still open and abounds in important unsolved problems; and to emphasize the importance of working toward a solution of the deepest, most difficult problems.

The Millennium Prize Problems gives the official description of each of the seven problems and the rules governing the prizes. It also contains an essay by Jeremy Gray on the history of prize problems in mathematics.

The seven Millennium Prize Problems range from the oldest, the Riemann Hypothesis, a problem in number theory stated in 1859, to the youngest, the **P** versus **NP** problem, a problem in theoretical computer science stated in 1971. Informal descriptions are given below. See the individual articles in the book for official statements and background material. See also www.claymath.org/millennium

The Birch and Swinnerton-Dyer Conjecture: Let C be an elliptic curve over the rational numbers. The order to which its L function vanishes at $s = 1$ is the rank of the group of rational points.

The Hodge Conjecture: A rational cohomology class of type (p,p) on a projective algebraic manifold is represented by a rational sum of algebraic subvarieties.

The Navier–Stokes Equation. Show that the Navier–Stokes equations on Euclidean 3-space have a unique, smooth, finite energy solution for all time greater than or equal to zero, given smooth, divergence-free, initial conditions which "decay rapidly at large distances." Alternatively, show that there is no such solution.

Poincaré Conjecture: a closed, compact, simply connected three-dimensional manifold is homeomorphic to the three-dimensional sphere.

P versus **NP** Problem: If a proposed solution to a problem can always be verified in polynomial time, can a solution always be found in polynomial time?

Riemann Hypothesis: Let s be a zero of the Riemann zeta function different from one of the trivial zeros $z = -2, -4, -6, \ldots$. Then the real part of s is 1/2.

Quantum Yang–Mills Theory: Prove that for any compact simple group G, quantum Yang–Mills theory exists on 4-space and has a mass gap $\Delta > 0$.

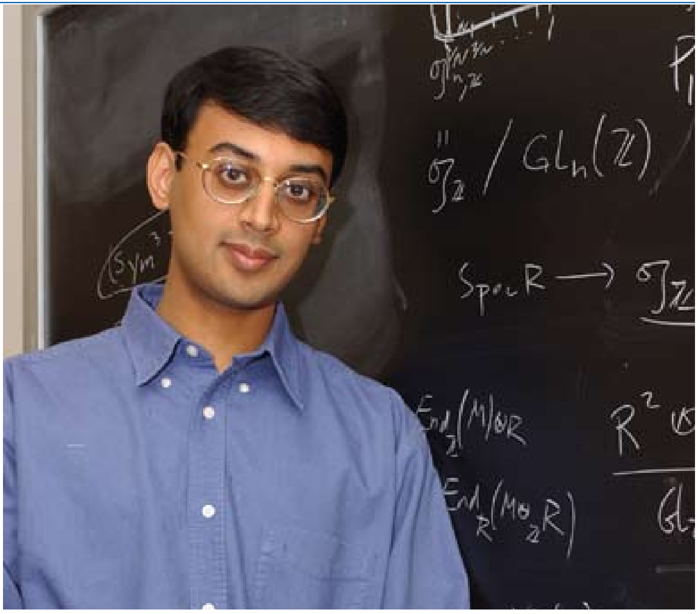
2005 Clay Research Awards

The Clay Mathematics Institute presents the Clay Research Awards annually to recognize contemporary breakthroughs in mathematical research. Awardees receive significant support for their research for one year and the bronze sculpture "Figureight Knot Complement vii/ CMI" by sculptor Helaman Ferguson. Since 2000, the practice has been to choose one senior and one junior mathematician for the award. Fourteen mathematicians have received the award since its inception in 1999. The first award was presented to Andrew Wiles of Princeton University at the Institute's annual meeting at MIT on May 10, 1999. The most recent awards were presented to Manjul Bhargava and Nils Dencker.

The 2005 Clay Research Awards were given at the Institute's Annual Meeting, held at Oxford University on October 11, 2005 in conjunction with a conference it organized on the publication of a digital edition of the earliest extant manuscript of Euclid's Elements (888 AD). At the meeting Andrew Wiles gave a general talk on solving equations. Special thanks to Oxford University, the Mathematical Institute, and the Said Business School for their collaboration and support in hosting the event.

CMI recognizes **Manjul Bhargava** for his discovery of new composition laws for quadratic forms and for his work on the average size of ideal class groups. The field of composition laws had lain dormant for 200 years after the pioneering work of C.F. Gauss. The laws discovered by Bhargava were a complete surprise and led him to another major breakthrough, namely, counting the number of quartic and quintic number fields with a given discriminant. The ideal class group is an object of fundamental importance in number theory. Conjectures of Cohen and Lenstra suggested a proposed answer to this problem. However, there was not a single proven case before Bhargava's work which solved the problem for the 2-part of the class groups of cubic fields. Curiously, in that case, the numerical evidence had led people to doubt the Cohen–Lenstra heuristics.

In his talk on higher composition laws at the awards ceremony in Oxford, Bhargava described how his fascination with Rubik's cube led to his discovery of the new composition laws.



Manjul Bhargava. Denise Applewhite, Office of Communications, Princeton University

As part of Bhargava's research award, the Institute is providing support over a two-year period to the Ramanujan Mathematical Society (RMS). The RMS has established the Clay–Bhargava Fellowship to further the research work of talented graduate students. See www.ramanujanmathsociety.org

Manjul Bhargava was a Clay Research Fellow from 2000 through 2005. In addition to mathematics, his interests include Indian classical music. See www.claymath.org/fas/research_fellows/Bhargava

Manjul Bhargava was born in Hamilton, Ontario, but spent most of his early years in Long Island, New York. He received his AB in Mathematics *summa cum laude* from Harvard University in 1996 and his PhD from Princeton University in 2001 under the direction of Andrew Wiles. After brief visiting positions at the Institute for Advanced Study and Harvard University, he joined the faculty of Princeton University in 2003 as a Professor of Mathematics. Bhargava's primary research interests are in number theory, representation theory, and algebraic geometry, Bhargava has received numerous awards and honors, including three Derek Bok Awards for Excellence in Teaching (1993–95), the Hoopes Prize for Excellence in Scholarly Work and Research from Harvard University (1996), the AMS–MAA–SIAM Morgan Prize for Outstanding Undergraduate Research in Mathematics (1997), the MAA Merten M. Hasse Prize for Exposition (2003), a Packard Foundation Fellowship in Science and Engineering (2004), and the AMS Blumenthal Award for the Advancement of Pure Mathematics (2005). Bhargava was also the Clay Mathematics Institute's first Fellow (2000–05). He was named one of Popular Science magazine's "Brilliant 10" in 2002. Bhargava is an accomplished tabla player.

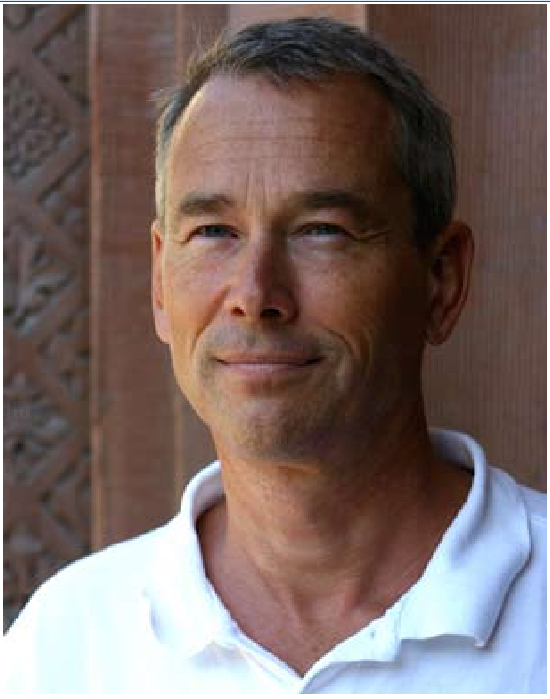
For more information, see
www.claymath.org/research_award

Recognizing achievement

CMI recognizes **Nils Dencker** for his complete resolution of a conjecture made by F. Trèves and L. Nirenberg in 1970. This conjecture posits an essentially geometric necessary and sufficient condition, *Psi*, for a pseudo-differential operator of principal type to be locally solvable, i.e., for the equation $Pu = f$ to have local solutions given a finite number of conditions on f. Dencker's work provides a full mathematical understanding of the surprising discovery by Hans Lewy in 1957 that there exists a linear partial differential operator – a one-term, third-order perturbation of the Cauchy-Riemann operator – which is not locally solvable in this sense. The necessity of condition *Psi* was shown for operators in dimension 2 by R. Moyer in 1978 and in general by L. Hörmander in 1981. The sufficiency of the condition had resisted many previous attacks.

Nils Dencker was born in 1953 in Lund, Sweden. He received his PhD from Lund University in 1981 under the direction of Lars Hörmander. After spending 1981–1983 as a C.L.E. Moore instructor at MIT, he returned to Lund University where he became Director of Studies in the Department of Mathematics from 2001–2003. Dencker received the Gårding prize in 2003 from the Royal Physiographic Society in Lund. He is currently the Vice Chairman of the Swedish Mathematical Society.

Dencker's research interests are in the microlocal analysis of partial differential equations and calculus of pseudo-differential operators. He has studied the propagation of polarization (vector-valued singularities) for systems of partial differential equations, e.g., in double refraction. Dencker has also studied the pseudospectra (spectral instability) of semi-classical partial differential equations and the solvability of partial differential equations.



Nils Dencker

Past Recipients

2004 Ben Green (Bristol University): Arithmetic progressions in the primes.

Gérard Laumon & Bao-Châu Ngô (Université de Paris-Sud XI, Orsay): Fundamental lemma in the Langlands program.

2003 Richard Hamilton (Columbia University): Ricci flow.

Terence Tao (UCLA): Optimal restriction theorems in Fourier analysis, Horn's conjecture.

2002 Oded Schramm (Microsoft Research): Stochastic Loewner equation.

Manindra Agrawal (Indian Institute of Technology, Kanpur, India): Primality can be decided in polynomial time.

2001 Edward Witten (Institute for Advanced Study, Princeton): Fundamental contributions to both mathematics and physics.

Stanislav Smirnov (Royal Institute of Technology, Stockholm): Existence of the scaling limit of two-dimensional percolation, verification of Cardy's conjectures.

2000 Alain Connes (Collège de France): For revolutionizing the field of operator algebras, for inventing modern non-commutative geometry.

Laurent Lafforgue (IHES): Work on the Langlands program.

1999 Andrew Wiles (Princeton University): For research in number theory.

Summary of 2005 Research Activities

The activities of CMI researchers and research programs are sketched below. Researchers and programs are selected by the Scientific Advisory Board (see inside back cover).

Clay Research Fellows

Bo'az Klartag graduated from Tel Aviv University. His three-year appointment began in September 2005. He is based at the Institute for Advanced Study in Princeton, New Jersey.

David Speyer graduated from UC Berkeley and is working at the University of Michigan. He has a five-year appointment.

Klartag and Speyer joined current research fellows Manjul Bhargava (Princeton University), Daniel Biss (University of Chicago), Alexei Borodin (Caltech), Maria Chudnovsky (Princeton), Sergei Gukov (Caltech), Elon Lindenstrauss (Princeton), Ciprian Manolescu (Columbia University), Maryam Mirzakhani (Princeton University), Ben Green (MIT), András Vasy (Stanford) and Akshay Venkatesh (Princeton).

Book Fellows

Appointed in 2005 were Steven Finch (Boston University), who worked on a second volume of "Mathematical Constants," Grigory Mikhalkin (Toronto & Utah), who is writing a book on "Tropical Geometry and Amoebas," and John Morgan (Columbia University) and Gang Tian (Princeton & MIT), who are collaborating on a monograph on Ricci flow and Perelman's work.

Senior Scholars

Eric Zaslow (Northwestern University), November 1, 2004–July 14, 2005. Fields Institute thematic program on the Geometry of String Theory.

David Donoho (Stanford University), January 1–March 31, 2005. MSRI program on Mathematical, Computational and Statistical Aspects of Image Analysis.

Peter Winkler (Dartmouth College), January 1–

Barry Mazur's Clay Public Lecture "Are there unsolved problems about numbers?"

March 31, 2005. MSRI program on Probability, Algorithms and Statistical Physics.

Simon Levin (Princeton), June 26–July 16, 2005. PCMI program on Mathematical Biology.

Charles Peskin (Courant Institute, NYU), June 26–July 16, 2005. PCMI program on Mathematical Biology.

David Morrison (Duke), August 1–December 16, 2005. KITP program on Mathematical Structures in String Theory.

Robert Bartnik (Canberra), August 8–December 23, 2005. Newton Institute program on Global Problems in Mathematical Relativity.

Nigel Hitchin (Oxford), September 1–30, 2005. Kavli Institute program on String Theory.

Leo Kadanoff (Chicago), September 1–December 31, 2005. Fields Institute program on Renormalization and Universality in Mathematics and Mathematical Physics.

Research Scholars

Aderemi Kuku (ICTP, Trieste). August 15, 2004–March 31 2005 at MSRI and Ohio State.

Hiroshi Saito (Nagoya University). February 26–March 26, 2005 at Université Paris-Sud XI, Orsay.

Wandera Ogana (University of Nairobi). March 20–September 30, 2005 at Ohio State University.

Minoru Wakimoto (Kyushu University). April 1–May 31, 2005 at MIT.

Manfred Einsiedler (Ohio State University). July 1– December 31, 2005 at Princeton University.

Haruzo Hida (UCLA). September 1–30, 2005 at CRM, Montreal.

Peter Schneider (Münster). September 1–30, 2005 at CRM, Montreal.

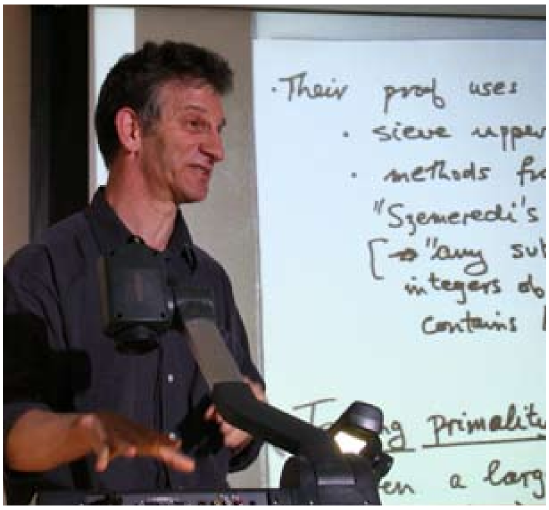
Wolfgang Ziller (U. Penn). September 1, 2005–June 30, 2006 at IMPA, Brazil.

Yaroslav Vorobets (Pidstryhach Institute of Applied Problems of Mechanics and Mathematics at NASU, Ukraine). September 1, 2005–August 31, 2006 at Texas A&M University.

Jacques Tilouine (Institut Galilée, Université Paris-Nord XIII). November 1–30, 2005 at CRM, Montreal.

Liftoff Fellows

CMI appointed seventeen Liftoff Fellows for the summer of 2005. Clay Liftoff Fellows are recent Ph.D. recipients who receive one month of summer salary and travel funds before their first academic position. See www.claymath.org/liftoff



Peter Sarnak's colloquium talk on prime numbers during the 2005 Clay Research Academy

Collaborators Tom Sanders and Ben Green at the Clay Mathematics Institute

Conferences Organized and Supported

Chern Memorial Conference at CIMAT, November 17–19, CIMAT, Guanajuato, Mexico.

Lie Groups, Representations, and Discrete Mathematics, November 14–18, IAS, Princeton.

Algebraic Statistics and Computational Biology, November 12–14, Clay Mathematics Institute.

Analytic and Stochastic Fluid Dynamics, October 10–14, MSRI, Berkeley.

Cluster-Polyfold Setup for Lagrangian Floer Homology, October 10–14, IAS, Princeton.

Euclid and his Heritage, October 6–8, St. Catherine's College, Oxford University.

Mathematical Structures in String Theory August 1–December 16, Kavli Institute for Theoretical Physics.

Summer Institute in Algebraic Geometry July 25–August 12, University of Washington, Seattle.

Geometric and Topological Methods for Quantum Field Theory, July 11–29, 2005, Villa de Leyva, Colombia.

Rigidity, Dynamics and Group Actions July 9 - 14, Banff International Research Station, Vancouver BC, Canada.

Rigidity, Dynamics and Group Actions
July 9–14, Banff International Research Station, Vancouver BC, Canada.

Gauss-Dirichlet Conference, June 20–24, Georg-August-Universität, Göttingen, Germany.

Representation Theory, Geometry and Automorphic Forms, June 5–9, Tel Aviv University, Israel.

CMI Summer School on Ricci Flow, 3-Manifolds and Geometry, June 20–July 15, MSRI, Berkeley. See page 15 for more information.

CMI Workshop

Emergent Applications of the Goodwillie Calculus, March 11–13.

Clay Research Academy

In April, CMI held its fourth high school workshop, the Clay Mathematics Research Academy, which brought together some of the world's most mathematically talented high-school students for nine days of study and collaboration with leading researchers. Richard Stanley (MIT) led a program on generating functions and bijective proofs, and Pavel Etingof (MIT) led a program on group representations and their applications. The Academy Colloquium Series attracted much interest (drawing an average crowd of over 100). The speakers and topics were as follows:

Impossibility theorems on integration in elementary terms; Brian Conrad (Michigan).

The multiple facettes of the associahedron; Jean-Louis Loday (Strasbourg, France).

E6, E7, E8; Nigel Hitchin (Oxford, UK).

Bott Periodicity in Topological, Algebraic and Hermitian K-Theory (Part I); Max Karoubi (Paris 7, France).

Bott Periodicity in Topological, Algebraic and Hermitian K-Theory (Part II); Max Karoubi (Paris 7, France).

Sizes and Scales in the Sub-atomic World; Gerard 't Hooft (ITP, Utrecht).

Prime Numbers; Peter Sarnak (Princeton).

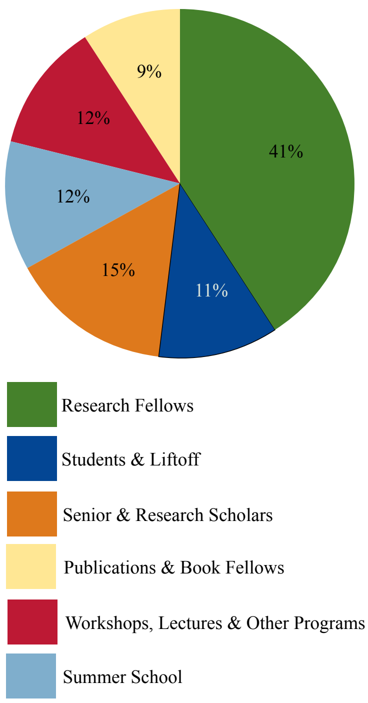
Quivers in Algebra, Geometry, and Representation Theory; Victor Ginzburg (Chicago).

Notes and slides for the talks are available online at www.claymath.org/programs/outreach/academy/colloquium2005.php

Program Allocation

Estimated number of persons supported by CMI in selected scientific programs for calendar year 2005:

Research Fellows, Research Awardees, Senior Scholars, Research Scholars, Book Fellows and Public Lecturers	41
Summer School participants and faculty	120
Research Academy and Student Programs, participants and faculty	100
CMI Workshops, Liftoff program	50
Participants attending joint programs and the Independent University of Moscow	> 1000



Research Expenses for Fiscal Year 2005
(comparative allocations change annually based on scientific merit)

Collected Works

James Arthur

James Arthur Archive

This website collects the author's complete published work in an easily accessible set of searchable PDF files. See www.claymath.org/cw/arthur. The archive is the work of Vida Salahi at the Clay Mathematics Institute, with help from Bill Casselman at the University of British Columbia. James Arthur will add comments to many of the articles in the near future. CMI wishes to thank all the publishers for permission to scan the papers.

James Arthur was born on May 18, 1944. He attended the University of Toronto as an undergraduate, and received his Ph.D. at Yale University in 1970, where his advisor was Robert Langlands. He has been a University Professor at the University of Toronto since 1987.

Almost all of Arthur's professional career has been dedicated to exploring the analogue for general reductive groups of the trace formula for SL_2 first proved by Selberg in the mid 1950s. This has proved to be enormously complex in its details, but also extraordinarily fruitful in its applications.

Raoul Bott Library

The Clay Mathematics Institute is greatly honored to receive the mathematical library of Raoul Bott, who passed away on December 20, 2005. Professor Bott was a deep and creative thinker, one of the great mathematical minds of the 20th century. Few have spoken more eloquently of his life and work than his Harvard colleague Cliff Taubes. Here are some of his words:

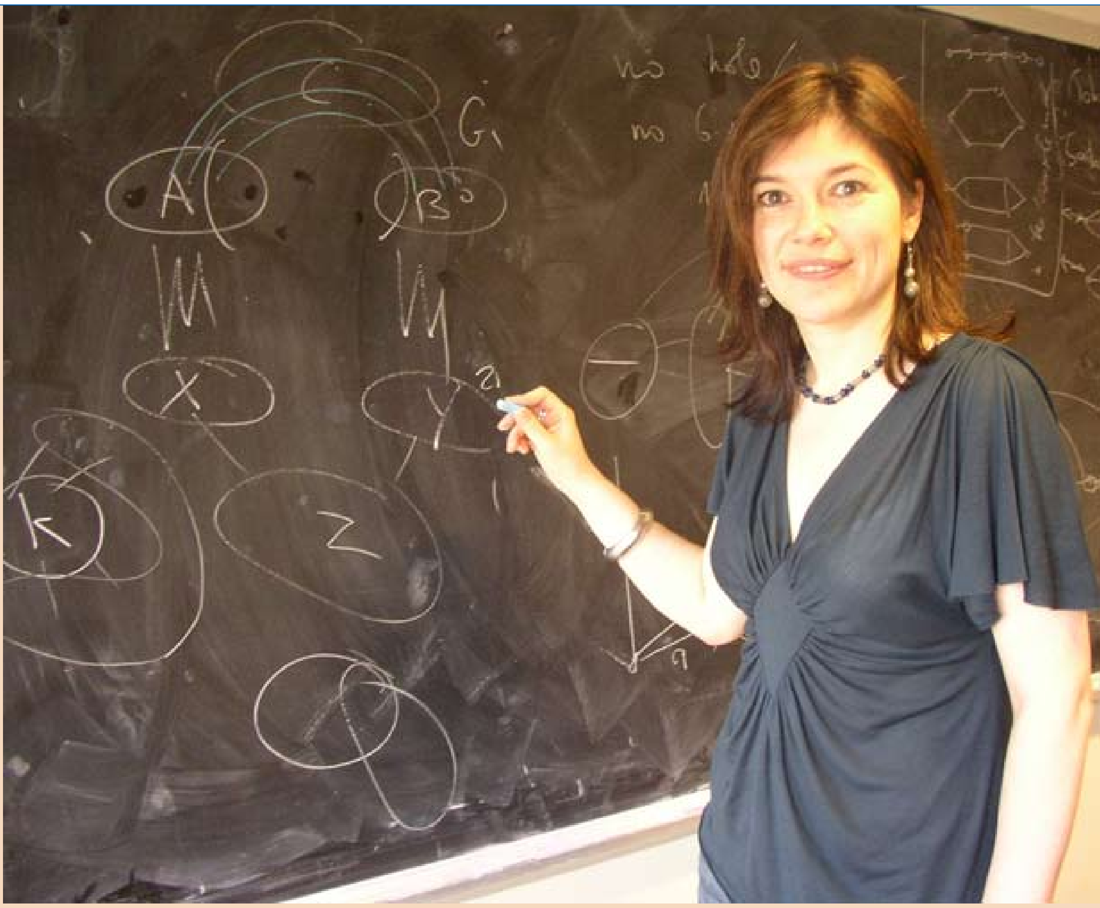
His theorems were fantastic, but there are people with fantastic theorems who are not loved the way he was loved. Everyone considered him a father figure. He was just such a gentleman and so gregarious. He loved to laugh, he loved life. He taught us to look for beauty and art in everything.

Raoul worked very hard to say it right, to say it as cleanly as possible. His papers are gems – not incompressible, nor jargon-filled. They were works of art.

Raoul Bott with Michael Atiyah in his office at the Mathematical Institute at Oxford University in the early 1980s

CMI profile

Interview with Research Fellow Maria Chudnovsky



Maria Chudnovsky: Photo by Dona Vukson, 2006

Maria Chudnovsky received a BA and MSc from the Technion and a PhD from Princeton University in 2003. Currently she is a Clay Research Fellow and an assistant professor at Princeton University. Her research interests are in discrete mathematics and, in particular, graph theory. Recently, she was part of a team of four researchers that proved the Strong Perfect Graph Theorem, a forty-year-old conjecture that had been, arguably, the central open problem in graph theory. For this work, she was awarded the Ostrowski foundation research stipend.

What first drew you to mathematics? What are some of your earliest memories of mathematics?

What I have always liked about mathematics is that everything can be explained from "first principles." If you understand something, you really understand it all the way through; it's all in your head, and you can always go and check it.

My first memories of mathematics are probably of elementary or junior high school geometry. That must have been the first time I saw a proof – I realized for the first time that there is no place for any further discussion. Another memory was when a teacher in junior high school mentioned, in passing, that there was more than one kind of infinity. I knew I had to grow up and find out what he meant.

Could you talk about your mathematical education in Russia and Israel? What experiences and people were especially influential?

I went to elementary school and junior high in Russia and to high school in Israel. I have been very fortunate in my mathematical education. In Russia I went to a special school that emphasized the study of mathematics. Back then it was called School #30, in St Petersburg. It is probably called something different now. In Israel I was in a program that followed the "Columbia System." As far as I know, this is a program of study that was developed at Columbia University. The idea was to introduce some pretty abstract notions quite early on. For example, we first learned what a group was in 9th grade. In both Russia and Israel, I was very lucky to have great teachers, teachers who had passion and enthusiasm for mathematics, and who made us, the students, believe that what we were learning in that class was the most interesting thing in the world.

In both Russia and Israel, I was very lucky to have great teachers, teachers who had passion and enthusiasm for mathematics, and who made us, the students, believe that what we were learning in that class was the most interesting thing in the world.

Did you have a mentor? Who helped you develop your interest in mathematics, and how?

Here I must mention a "math circle" I went to in 11th and 12th grade. I lived in Haifa, and a friend from school told me that on Thursday afternoons one could go to the Technion and take this informal class run by mathematics graduate students. It was an absolutely amazing experience! Sometimes we would think about problems, other times the teachers would tell us a simplified version of a lecture that they themselves had heard a few days earlier. Again, we all felt that nothing out there could even compare to what we were doing. That was when I decided that I would major in math in college. As I studied more mathematics over the next ten years, the problems got harder, the lectures got more complicated, but the feeling that there is nothing better I could possibly do with my time is still there. On a slightly more serious note, I should, of course, mention and thank the people who guided my studies in college and throughout graduate school – professors Ron Aharoni and Abraham Berman at the Technion, and professor Paul Seymour at Princeton.

You served in the Israeli military from 1996 to 1999. Did the military make use of your mathematical talent?

One of the greatest things about mathematics is that it teaches you to think clearly and to be very critical of the logic of your arguments. My experience in the military showed that this skill is valuable in other areas.

How would you compare your mathematical experiences in Russia, Israel, and the US?

I am not sure I can answer this question, as I have spent different periods of my life in these three countries. Thus I did not really get a chance to make a comparison.

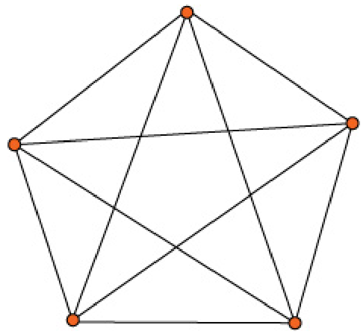
What attracted you to combinatorics and the particular problems you have studied?

Most of these problems are very easy to describe, but they do not have simple solutions. In fact, the reason for the answer being one way or another is often quite deep. In order to find the solution, one needs to uncover layers of phenomena that seem to have nothing to do with the original question.

Can you describe your research in accessible terms? Does it have applications to other areas?

One of the greatest things about mathematics is that it teaches you to think clearly and to be very critical of the logic of your arguments. My experience in the military showed that this skill is valuable in many other areas.

I study objects called "graphs." A graph is a collection of points, called "vertices," such that some pairs of points are "adjacent" and others are not. In many cases a set of data can be conveniently viewed as a graph: for example, take a graph whose vertices are train stations, and two vertices are adjacent if and only if there is a direct train running between the two stops. Finding a quick way to get between two stations can be formulated as an abstract problem about graphs. The Internet can also be viewed as a graph. What I do is study properties that graphs



Complete graph on five vertices

have. While I doubt that there are any immediate applications of what I do to any practical problems, graph theory in general is a useful tool in computer science and operations research.

What research problems and areas are you likely to explore in the future?

So far most of my work has been on families of graphs defined by the absence of a certain substructure (called an "induced subgraph"). I would very much like to know if there is a general theory of "excluded induced subgraphs," meaning that there are some phenomena that happen in graphs no matter what specific induced subgraph you exclude.

How has the Clay Fellowship made a difference for you?

In many ways! First of all, it allows me to travel to conferences and to discuss my work with other people. In my opinion this is one of the most important ingredients of the work of a researcher. Also, my fellowship is for five years, and it is really great to have the kind of stability that allows me to work on longer term projects, and not worry so much about the immediate output. This is something that most people do not have right out of graduate school, and I value it very much. Finally, the fact that Clay Fellows get to choose how much teaching they want to do, is, of course, a big benefit. To summarize, when I was applying for jobs after graduate school, being a Clay Fellow seemed like a perfect arrangement, and now I know that it really is.

What advice would you give to young people starting out in math (i.e., high school students and young researchers)?

I really do not like giving advice, especially to such a large and heterogeneous group of people... But maybe this – if you think math is what you want to do, give it a chance.

How do you think mathematics benefits culture and society?

Mathematics teaches us to think abstractly, and as time goes on this ability becomes more and more commonplace in society in general. In my view, this is the biggest contribution of mathematics to culture.

Please tell us about things you enjoy when not doing mathematics.

I think mathematics teaches us to think abstractly, and as time goes on this ability becomes more and more commonplace in society in general. In my view, this is the biggest contribution of mathematics to culture.

I have to say mathematics is the only "productive" thing that I do. I do not have any (other) hobbies with an actual "outcome," like painting, or playing an instrument. But there are many things that other people have already done that I enjoy. I read quite a bit. I like art. Right now I am trying to learn more about photography and photographers. I live in Princeton, which is only a one-hour drive from New York City. During my time here I have been trying to take as much advantage of this as I can.

Recent Research Articles

"The Strong Perfect Graph Theorem," with N. Robertson, P. Seymour, R. Thomas, to appear in the *Annals of Mathematics*.

"Recognizing Berge Graphs," with G. Cornuejols, X. Liu, P. Seymour, and K. Vuskovic, *Combinatorica* Vol. 25 (2005), 143-187.

"The Structure of Clawfree Graphs," with Paul Seymour, Surveys in Combinatorics 2005, London Math Soc Lecture Note Series, Vol. 327.

Can Biology Lead to New Theorems?

CAN BIOLOGY LEAD TO NEW THEOREMS?

BERND STURMFELS

Abstract. This article argues for an affirmative answer to the question in the title. In future interactions between mathematics and biology, both fields will contribute to each other, and, in particular, research in the life sciences will inspire new theorems in "pure" mathematics. This point is illustrated by a snapshot of four recent contributions from biology to geometry, combinatorics and algebra.

Much has been written about the importance of mathematics for research in the life sciences in the 21st century. Universities are eager to start initiatives aimed at promoting the interaction between the two fields, and the federally funded mathematics institutes (AIM, IMA, IPAM, MBI, MSRI, SAMSI) are outdoing each other in offering programs and workshops at the interface of mathematics and the life sciences. The Clay Mathematics Institute has had its share of such programs. For instance, in the summer of 2005, two leading experts, Charles Peskin and Simon Levin, served as Clay Senior Scholars in the *Mathematical Biology* program at the IAS/Park City Mathematics Institute (PCMI), and in November 2005, Lior Pachter, Seth Sullivant and the author organized a workshop on *Algebraic Statistics and Computational Biology* at the Clay Mathematics Institute in Cambridge.

Yet, as these ubiquitous initiatives and programs unfold, many mathematicians remain unconvinced, and some secretly hope that this "biology fad" will simply go away soon. They have not seen any substantive impact of quantitative biology in their area of expertise, and they rightfully ask: **where are the new theorems?**

In light of these persistent doubts, some long-term observers wonder whether anything has really changed in the twenty years since Gian-Carlo Rota wrote his widely quoted sentence, *"The lack of real contact between mathematics and biology is either a tragedy, a scandal, or a challenge, it is hard to decide which"* [16, page 2]. Of course, Rota was well aware of the long history of mathematics helping biology, such as the development of population genetics by Fisher, Hardy, Wright and others in the early 1900's. Nonetheless, Rota concluded that there was no "real contact".

But, quite recently, other voices have been heard. Some scholars have begun to argue that "real contact" means being equal partners, and that meaningful intellectual contributions can, in fact, flow in both directions. This optimistic vision is expressed succinctly in the title of J.E. Cohen's article [6]: *"Mathematics is biology's next microscope, only better; biology is mathematics' next physics, only better"*.

Physics remains the gold standard for mathematicians, as there has been "real contact" and mutual respect over a considerable period of time. Historically, mathematics has made many contributions to physics, and in the last twenty years there has been a payback beyond expectations. Many of the most exciting developments in current mathematics are a direct outgrowth of research in theoretical physics. Today's geometry and topology are unthinkable without string theory, mirror symmetry and quantum field theory. It is "obvious" that physics can lead to new theorems. Any colloquium organizer in a mathematics department who is concerned about low attendence can reliably fill the room by scheduling a leading physicist to speak. The June 2005 public lecture on *Physmatics* by Clay Senior Scholar Eric Zaslow sums up the situation as follows: *"The interplay between mathematics and physics has, in recent years, become so profound that the lines have been blurred. The two disciplines, long complementary, have begun a deep and fundamental relationship..."*.

Will biology ever be mathematics' next physics? In the future, will a theoretical biologist ever win a Fields medal? As unlikely as these possibilities seem, we do not know the answer to these questions. However, my recent interactions with computational biologists have convinced me that there is more potential in this regard than many mathematicians may be aware of. In what follows I wish to present a personal answer to the legitimate question: **where are the new theorems?**

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CMI Summer Schools

2007	Homogeneous flows, moduli spaces, and arithmetic De Giorgi Center, Pisa	
2006	Arithmetic Geometry Mathematisches Institut, Georg-August-Universität, Göttingen	
2005	Ricci Flow, 3-manifolds, and Geometry MSRI, Berkeley	
2004	Floer Homology, Gauge Theory, and Low-dimensional Topology Rényi Institute, Budapest	
2003	Harmonic Analysis, Trace Formula, and Shimura Varieties Fields Institute, Toronto	
2002	Geometry and String Theory Newton Institute, Cambridge UK	
2001	Minimal surfaces MSRI, Berkeley	
2000	Mirror Symmetry Pine Manor College, Boston	

The Clay Mathematics Institute has conducted a program of research summer schools since 2000. Designed for graduate students and PhDs within five years of their degree, the aim of the summer schools is to furnish a new generation of mathematicians with the knowledge and tools needed to work successfully in an active research area. Three introductory courses, each three weeks in duration, make up the core of a typical summer school. These are followed by one week of more advanced minicourses and individual talks. Size is limited to roughly 100 participants in order to promote interaction and contact. Venues change from year to year, and have ranged from Cambridge, Massachusetts to Pisa, Italy. The lectures from each school are published in the CMI–AMS proceedings series, usually within two years' time.

Summer Schools
www.claymath.org/programs/summer_school

Summer School Proceedings
www.claymath.org/publications

2006 Arithmetic Geometry Summer School in Göttingen

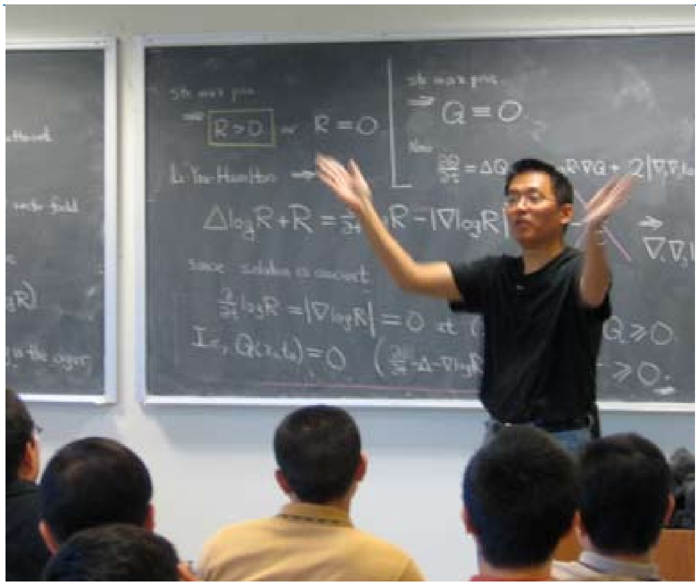
The 2006 summer school program will introduce participants to modern techniques and outstanding conjectures at the interface of number theory and algebraic geometry.

The main focus is the study of rational points on algebraic varieties over non-algebraically closed fields. Do such points exist? If not, can this fact be proven efficiently and algorithmically? When rational points do exist, are they finite in number, and can they be found effectively? When there are infinitely many rational points, how are they distributed?

For curves, a cohesive theory addressing these questions has emerged in the last few decades. Highlights include Faltings' finiteness theorem and Wiles' proof of Fermat's Last Theorem. Key techniques are drawn from the theory of elliptic curves, including modular curves and their parametrizations, Heegner points, and heights.

The arithmetic of higher-dimensional varieties is equally rich, offering a complex interplay of techniques including Shimura varieties, the minimal model program, moduli spaces of curves and maps, deformation theory, Galois cohomology, harmonic analysis, and automorphic functions. Many foundational questions about the structure of rational points remain open, however, and research tends to focus on properties of specific classes of varieties.

This summer school will offer three core courses (on curves, surfaces, and higher-dimensional varieties), supplemented by seminars on computational and algorithmic aspects of arithmetic geometry, and by mini-courses on more advanced topics.



Ben Chow's course on Ricci flow at the 2005 school

Summer School on Ricci Flow, 3-Manifolds, and Geometry

The 2005 school was held June 20–July 15 at the Mathematical Sciences Research Institute (MSRI) in Berkeley, California. Its aim was to give an in-depth introduction to the theory and applications of Ricci flow, beginning with Richard Hamilton's seminal 1982 paper and ending with a sketch of Perelman's claimed solution of the Poincaré conjecture.

Ben Chow gave a three-week-long course on Ricci flow, beginning with the background needed to understand Hamilton's theorem on 3-manifolds with positive Ricci curvature. This theorem states that Ricci flow transports an initial metric on a closed Riemannian manifold with positive Ricci curvature toward one that has constant positive sectional curvature; convergence is exponentially fast. Notes for Ben's course, partly based on the books *The Ricci flow: An introduction,* by Chow-Knopf and *Hamilton's Ricci flow,* by Chow-Lu-Ni, were posted on the web in advance of the school, giving participants a chance to prepare themselves. See www.claymath.org/programs/summer_school/2005/program.php

Bruce Kleiner and John Lott gave a three-week course on Perelman's work: Perelman's non-collapsing theorem, Perelman's reduced volume, the entropy functional, Kappa-ancient solutions and classification in three dimensions, analysis of the large curvature part of Ricci flow solutions, and applications to geometrization.

John Morgan and Jeff Cheeger gave a two-week course. Morgan spoke on geometrization: the eight three-dimensional geometries, prime decomposition of 3-manifolds, incompressible tori, Thurston's geometrization conjecture on 3-manifolds, and graph manifolds. Cheeger spoke on compactness theorems in Riemannian geometry, manifolds of nonnegative curvature, and Alexandrov spaces. Their lectures were followed by a one-week course on minimal surfaces by Tobias Colding, David Hoffman, and Gabriele La Nave.

The fourth week of the school was devoted to advanced courses that introduced participants to important problems of current research. Lectures were given by Jeff Cheeger, Bennett Chow, Tobias Colding, Richard Hamilton, David Hoffman, Bruce Kleiner, Gabriele La Nave, John Lott, Peng Lu, John Morgan, Andre Neves, Lei Ni, and Gang Tian.

The organizing committee for the school consisted of Gang Tian, John Lott, John Morgan, Bennett Chow, Tobias Colding, Jim Carlson, David Ellwood, and Hugo Rossi.

Ricci Flow Summer School participants

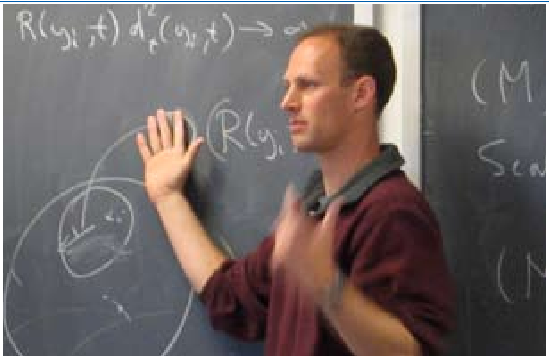
CMI Summer Schools

The Ricci flow equation is much like the heat equation, which was first intensively studied by Joseph Fourier in his "Théorie Analytique de la Chaleur" in 1822. While the heat equation governs the way temperature in a material body evolves over time, the Ricci flow equation governs the evolution of an object's geometry, that is, its metric. The heat equation is linear, and any roughness in the initial temperature distribution smooths out as time increases. If now my coffee is very hot and my martini is very cold, then (alas) both will be lukewarm in the not-too-distant future. The Ricci flow equation, however, is nonlinear. Thus, while smoothing of the geometry can occur – little bumps of curvature can spread and disappear – curvature can also concentrate in some regions and lead to the formation of singularities. For example, join two round spheres by a smooth neck. Ricci flow progressively narrows the neck, which then pinches off. Thus, the geometry of a manifold can undergo not only quantitative but also qualitative change.

There are very few natural equations that can describe the evolution of the geometry of a manifold. On one side must be the time derivative of the metric tensor, the quantity that defines geometry. On the other side must be a tensor of the same kind, constructed from the metric tensor. The simplest such tensor besides the metric tensor itself is the Ricci tensor. The equation

$$\frac{\partial g_{ij}}{\partial t} = -2R_{ij}$$

is the natural (and inspired) guess. This is the Ricci Flow Equation, formulated in Richard Hamilton's celebrated 1982 paper. In that paper Hamilton showed that a compact three-manifold of positive definite Ricci curvature evolves by Ricci flow (and rescaling) to a manifold of constant positive sectional curvature. If the manifold is also simply-connected, then it must be the three-sphere. Subsequently, Hamilton made a detailed study of the development of singularities under Ricci flow, and he introduced and studied the notion of Ricci flow with surgery to deal with singularity formation. Underlying this effort was the allure of a proof of the Poincaré conjecture



Bruce Kleiner's course on Perelman's work at the 2005 Ricci Flow School

and, more generally, of Thurston's geometrization conjecture.

The Poincaré conjecture characterizes the simplest of all closed three-manifolds – the three-sphere – as the only 3-manifold in which closed loops (think of little strings) can be shrunk to a point without breaking them or letting them move off the sphere. Thurston's geometrization conjecture asserts that all possible closed three-dimensional manifolds can be built in standard ways from basic building blocks which we understand; it includes the Poincaré conjecture as a special case.

Grisha Perelman, in a series of three papers posted on arXiv.org beginning in November of 2002, announced a solution of the Poincaré and the geometrization conjectures using Ricci flow. His papers contained a series of breakthroughs that allowed one to successfully deal with all types of singularity formation, with the problems encountered in Ricci flow with surgery, and with the formation of hyperbolic pieces as time tends to infinity. In particular, Pereleman's papers (listed below) gave a crucial argument to show that the set of surgery times is discrete.

Perelman's work set off a flurry of activity aimed at understanding and verifying his claims. Most of that work to date is available publically via the web page www.math.lsa.umich.edu/~lott/ricciflow/perelman.html, maintained by Bruce Kleiner and John Lott. Perelman's first two papers on Ricci flow were:

arXiv:math.DG/0303109 — November 11, 2002
The entropy formula for the Ricci flow and its geometric applications

arXiv:math.DG/0303109 — March 10, 2003
Ricci flow with surgery on three-manifolds

CMI Senior Scholars Program

The Clay Mathematics Institute initiated its Senior Scholars program in 2003 with the appointment of Richard Stanley (MIT) and Bernd Sturmfels (UC Berkeley). Stanley and Sturmfels were the mathematicians in residence for the IAS/PCMI 2004 summer program on Geometric Combinatorics in Park City, Utah, directed by Herb Clemens. Since then fifteen other senior scholars have been appointed in areas ranging from algebraic geometry and number theory to mathematical biology and string theory.

The aim of the Senior Scholars initiative is to increase opportunities for research institutes and universities to secure the early commitment of one or two senior mathematicians to a topical program for a significant length of time, generally three weeks to three months. Through both formal and informal interaction with other participants, from lectures and seminars to lunchtime discussions of mathematics, senior scholars play a central role in the program.

Senior scholars are encouraged to give a public lecture that reaches an audience wider than that of the research program in which they participate. Some of these lectures are available online at www.claymath.org/lectures: Richard Stanley, *Tilings*, Bernd Sturmfels and David Speyer, *Tropical Mathematics*, and Eric Zaslow, *Physmatics*. A video of Stanley's lecture is available on request. Please write to general@claymath.org.

Senior scholars are nominated by the sponsoring institute or university. For full consideration, nominations should be received by August 1. Nominations received at other times will be considered as funds permit. The host institution must not be in the nominees' area of residence. Nominations will be judged on the scientific impact of the program and the nominee's potential to contribute to it.

For more information, see www.claymath.org/senior_scholars

Bernd Sturmfels – PCMI 2003, Geometric Combinatorics.

Richard Stanley – PCMI 2003, Geometric Combinatorics.

Peter Winkler – MSRI 2004, Probability, Algorithms and Statistical Physics.

Robert Bartnik – Newton Institute 2005, Global Problems in Mathematical Relativity.

David Donoho – MSRI 2005, Mathematical, Computational and Statistical Aspects of Image Analysis.

Nigel Hitchin – KITP 2005, Mathematical Structures in String Theory.

Leo Kadanoff – Fields Institute 2005, Renormalization and Universality and Mathematics and Mathematical Physics.

Simon Levin – PCMI 2005, Mathematical Biology.

David Morrison – KITP 2005, Mathematical Structures in String Theory.

Charles Peskin – PCMI 2005, Mathematical Biology.

Jean-Louis Colliot-Thélène – MSRI 2006, Program on Rational and Integral Points on Higher Dimensional Varieties.

Yongbin Ruan – MSRI 2006, New Topological Methods in Physics.

Eric Zaslow – Fields Institute 2006, String Theory.

Yakov Eliashberg – PCMI 2006, Low-Dimensional Topology.

Robion Kirby – PCMI 2006, Low-Dimensional Topology.

John Milnor – PCMI 2006, Low-Dimensional Topology.

Peter Newstead – Tufts–BU 2006, Joint Semester on Vector Bundles.

Euclid and His Heritage Meeting

Speakers' Talks

From Euclid to Arethas
Alexander Jones (University of Toronto)

Rethinking the *Elements* – two thousand years of reflections on the foundations of geometry
Jeremy Gray (Open University)

The *Elements*: The transmission of the Greek text
Nigel Wilson (University of Oxford)

New technologies for the study of Euclid's *Elements*
Mark Schiefsky (Harvard University)

The mathematical legacy of Euclid's *Elements*
Robin Hartshorne (UC Berkeley)

Euclid's *Elements* in Hebrew
Tony Lévy (CNRS, Paris)

Interpreting Euclid – early and late
Bill Casselman (University of British Columbia)

Who started the Euclid myth?
Ian Mueller (University of Chicago)

Euclid's *Elements* in the Islamic world
Sonja Brentjes (Aga Khan University)

Clay mathematics: Euclid's Babylonian counterparts
Eleanor Robson (Cambridge University)

The Heiberg Edition of Euclid's *Elements*: an incorrect text or a false history of the text?
Bernard Vitrac (CNRS, Paris)

Euclid in Chinese... and in Manchu
Catherine Jami (CNRS, Paris)

The achievements and limitations of the theory of proportion in Euclid's *Elements* Book V
Christopher Zeeman (Warwick Mathematics Institute)

Speaker Christopher Zeeman

On October 7 and 8, 2005, the Clay Mathematics Institute held a conference, "Euclid and His Heritage," which brought classicists, historians, mathematicians and philosophers together to examine the transmission and influence of the founding document of mathematics. The occasion of the conference was the publication, for the first time, of a complete digital image of the oldest surviving manuscript of the *Elements* – a copy made by Stephen the Clerk for Arethas of Patrae, deacon and later bishop of Caesarea in Capadoccia, in Constantinople in the year 888 AD. The digital copy of the manuscript, which resides at the Bodleian Library, Oxford University, was produced by Octavo.com with support from the Clay Mathematics Institute.

The panel discussion at the conclusion of the conference

Speakers' Abstracts

Sonya Brentjes outlined current research on the transmission of the Arabic text of Euclid's *Elements*, including the remarkable variability of the text, differences with the Persian texts, and the scholarly as well as broader cultural relevance of the text and its transmitters.

Bill Casselman discussed the role of figures in geometry, beginning with one of the earliest extant, from Elephantine Island, dating from the 2nd century BC, ending with modern computer graphics, and discussing along the way the most complicated figure from ancient mathematics, the construction of the Icosahedron in Euclid's book XIII.

Jeremy Gray discussed the conceptual basis for Euclid's *Elements*: is the work about triangles, circles, etc., or is it about the notion of length and angle? Debates on the conceptual basis bear on fundamental problems such as that of the parallel postulate.

Robin Hartshorne spoke about the mathematical content of the *Elements* and of developments arising from this work: axiomatic foundations of geometry, the discovery of non-Euclidean geometries, the development of the real number system, and connections between geometry and modern algebra and analysis.

Catherine Jami discussed the long geometric tradition in China, the introduction of Euclid's work by the Jesuits in the late 16th century, and the appropriation of mathematics by Kangxi (1662–1722), second emperor of the Qing dynasty (1644–1911), for whom the *Elements* were rewritten, first in Manchu, then in Chinese.

Alexander Jones surveyed what we currently believe is the history of Euclid's *Elements* and its use in the Greek-speaking world during its first twelve centuries, starting with the elusive Euclid himself.

Tony Lévy discussed the Hebrew transmission of the Euclidean text: four different versions translated from Arabic sources, including that of Moses in Tibbon, completed in Provence in 1270, as well as Avicenna's (Ibn Sina) Foundations of Geometry, a shortened Arabic version of Euclid's *Elements*.

Ian Mueller spoke about the claims of indubitability (or certainty) in sixteenth-century discussions of mathematics and the role of the work of Proclus and Averroes in these discussions. An important historical factor was a misunderstanding of the Greek word for exactness or precision, *akribeia*.

Eleanor Robson reminded us that Euclid's contemporaries in Hellenistic Babylon were heirs to a mathematical tradition at least as ancient as Euclid's is for us today. Robinson explored the lives, works, and motives of the Babylonian mathematicians to better understand the extraordinary nature of Euclid's achievement in the context of his time.

Mark Schiefsky explored contributions of information technology to the study of Euclid's *Elements*, e.g., linking of electronic versions of the text to online manuscript images; linguistic technology for the analysis of parallel versions in different languages; mapping of deductive structures and their visual representation.

Bernard Vitrac recalled that since the 1990s, the adequacy of Heiberg's work establishing what is still the latest critical edition of the Greek text of the *Elements* has been questioned. In light of this, Vitrac discussed the contributions of Arabic and Arabo-Latin texts to the debate.

Nigel Wilson spoke on how Euclid's *Elements* was transmitted through the long period in which handwritten copies were the only means of preserving a text. Wilson illustrated his remarks with examples taken from the Bodleian Library Manuscript (888 AD) and some of the marginal notes therein written by medieval scribes.

Christopher Zeeman discussed the mathematics of Book V of the *Elements*, which is an exposition of the work of Eudoxus. Zeeman noted that in Euclid's framework one could not define the ratio of two ratios. However, the introduction of a new axiom for magnitudes, related to the Archimedean axiom but excluding infinitesimals, resolves the problem.

Appointments & Honors

Crick–Clay Professorship

In 2005 Landon and Lavinia Clay provided funding for a position at Cold Spring Harbor Laboratory, the Crick–Clay Professorship of Mathematics. The position has two aims: to honor the memory and achievements of Sir Francis Crick and to promote fruitful interaction between biology and mathematics.

The first person to hold the Crick–Clay Professorship is Dr. Carlos Brody. Brody, a native of Mexico, received a BA in physics from Oxford University in 1988, MSc from Edinburgh University in Artificial Intelligence in 1990, and a PhD in Computation and Neural Systems from Caltech in 1997. Among his honors is a Sloan Fellowship (2004–2005). Since 2001 he has been on the faculty of the Watson School of Biological Sciences at Cold Spring Harbor Laboratories.

Brody's current work centers on mathematical models of short-term memory and decision-making. In experimental psychology, short-term memory and decision-making are often studied with a task form known as "two stimulus-interval discrimination": Subjects are presented with a first stimulus (which we shall call "A"); a brief delay of a few seconds ensues; subjects are presented with a second stimulus ("B"); finally, subjects are asked to report the outcome of a binary decision based on the comparison of the two e.g., was A greater than B? Yes or No. In order to carry out this task correctly, subjects must use short-term memory to remember A through the delay between the first and the second stimulus. It is thought that the neural substrate of this short-term memory is in the form of a pattern of neural activity that is stable throughout the memory period. Different patterns of activity correspond to different remembered values of A. From a dynamical-systems perspective, to each value of A there corresponds a stable point. If there is a continuum of possible A values (for example, when the stimulus is the frequency of a pure tone), then the neural dynamics must implement, or approximate, a continuum of stable points – a "line attractor." In collaboration with experimental researchers who record from brains of animals trained to do two stimulus-interval discrimination tasks, the questions we ask are these: How can neural systems form such line attractors? In general, line attractors are fragile to perturbations. How can robustness be achieved? And, how can information stored in line attractors be used for decision-making?

2005 Olympiad Scholar

Sherry Gong, a 10th grade student at Phillips Exeter Academy, was named the 2005 Clay Olympiad Scholar at a ceremony in Washington, DC, on June 27, 2005.

The Clay Olympiad Scholar Award recognizes the most original solution to a problem on the US American Mathematics Olympiad (USAMO). It consists of a commemorative plaque, a cash award to the recipient, and a cash award to the recipient's school. The award is presented each year at the official awards dinnner for the USAMO, held in June in Washington, DC, at the State Department Ballroom.

2005 Olympiad Scholar – continued

Sherry Gong, daughter of Guhua Gong and Liangqing Li of San Juan, Puerto Rico, attended schools in Puerto Rico until 2005 when she enrolled at Philips Exeter Academy in Exeter, New Hampshire. Sherry attended a mathematics olympiad for the first time when she was in the sixth grade — the 3rd Olympiada Matematica de Centroamerica y el Caribe. There Sherry received a silver medal and a special award for the most original solution. It was the first such award in the history of that olympiad. Sherry received a silver medal the next year at the same olympiad. In 2003, she received a gold medal at the XVIII Olympiada Iberoamericana de Matematicas. She also received a bronze medal in the 44th IMO (2003) and a silver medal in the 45th IMO (2004).

In addition to mathematics, Sherry is interested in physics and computer programming. She won a position in the 24-member USA Physics Olympiad Team (2005). Sherry enjoys seeing the connection between physics and mathematics, and she likes to find her own solutions when given a math or physics problem.

Sherry won the State Championship for the Geo Bee and represented Puerto Rico in the National Geo Bee in Washington, DC (2002). She also likes karate, poetry, and reading.

The Prix Fermat Junior

Clay Research Academy student Igor Kortchemski won the Prix Fermat Junior for the work he began at the Academy. See http://shadowlord.free.fr/articles/fr_goodsf4f1.pdf

Harvard Traveling Fellow Jonathan Bloom

Jonathan Bloom graduated from Harvard College in 2004 with high honors in mathematics and a Harvard fellowship to travel the world for one year. He used this opportunity to examine how mathematics is taught and learned in other cultures, guided by the belief that, by looking abroad, those wishing to improve math education in the United States can better identify and evaluate their own assumptions.

With digital video equipment, software, and support provided by CMI, Jonathan began his travels at the 10th International Congress on Mathematical Education in Denmark and the 45th International Mathematics Olympiad in Greece. He then journeyed through Botswana, Israel, Thailand, Singapore, Japan, Australia, New Zealand, and Brazil. Along the way, Jonathan recorded primary, secondary, and teacher-education classes as well as interviews with students, teachers, mathematicians, and leaders in education. He also spent three months as a secondary school teacher in Botswana, where he found his students to be universally adept at arithmetic (few calculators!) but often challenged by more algebraic thinking.

Throughout the year, Jonathan got to know many mathematicians who, in addition to doing important research, were dedicated to improving the quality of math education in their countries. Inspired by their passion for both mathematics and teaching, he will begin the doctoral program in math at Columbia University in September, specializing in topology and geometry. He hopes to embark on an academic career of research, teaching, and direct involvement in math education.

Can Biology Lead to New Theorems? (continued)

I shall present four theorems which were inspired by biology. These theorems are in algebra, geometry and combinatorics, my own areas of expertise. I leave it to others to discuss biology-inspired results in dynamical systems and partial differential equations. Before embarking on the technical part of this article, the following disclaimer must be made: the mathematics presented below is just a tiny first step. The objects and results are certainly not as deep and important as those in Zaslow's lecture on Physmatics. But then, Rome was not built in a day.

We start our technical discussion with a contribution made by evolutionary biology to the study of metric spaces. This is part of a larger theory developed by Andreas Dress and his collaborators [2, 9, 10]. A *finite metric space* is a symmetric $n \times n$-matrix $D = (d_{ij})$ whose entries are non-negative ($d_{ij} = d_{ij} \geq 0$), zero on the diagonal ($d_{ii} = 0$), and satisfy the triangle inequalities ($d_{ik} \leq d_{ij} + d_{jk}$). Each metric space D on $\{1, 2, \ldots, n\}$ is a point in $\mathbb{R}^{\binom{n}{2}}$. The set of all such metrics is a full-dimensional convex polyhedral cone in $\mathbb{R}^{\binom{n}{2}}$, known as the *metric cone* [8].

With every point D in the metric cone one associates the convex polyhedron

$$P_D \; = \; \bigl\{ x \in \mathbb{R}^n \, : \, x_i + x_j \; \geq \; d_{ij} \;\text{ for all } i, j \bigr\}.$$

If $D_1, \ldots, D_k$ are metric spaces then $D_1 + \cdots + D_k$ is a metric space as well, and

$$P_{D_1 + D_2 + \cdots + D_k} \supseteq P_{D_1} + P_{D_2} + \cdots + P_{D_k}.$$

If this inclusion of polyhedra is an equality then we say that the sum $D_1 + D_2 + \cdots + D_k$ is *coherent*. A *split* is a pair (α, β) of disjoint non-empty subsets of $\{1, \ldots, n\}$ such that $\alpha \cup \beta = \{1, \ldots, n\}$. Each split (α, β) defines a *split metric* $D^{\alpha,\beta}$ as follows:

$$D^{\alpha,\beta}_{ij} = 0 \;\text{ if }\; \{i, j\} \subseteq \alpha \text{ or } \{i, j\} \subseteq \beta, \quad \text{ and } \; D^{\alpha,\beta}_{ij} = 1 \;\text{ otherwise.}$$

The polyhedron $P_{D^{\alpha,\beta}}$, which represents a split metric $D^{\alpha,\beta}$, has precisely one bounded edge, and its two vertices are the zero-one incidence vectors of α and β. A metric D is called *split-prime* if it cannot be decomposed into a coherent sum of a positive multiple of a split metric and another metric. The smallest example of a split-prime metric has $n = 5$, and it is given by the distances among the nodes in the complete bipartite graph $K_{2,3}$.

Theorem 1. (Dress-Bandelt Split Decomposition [2]**)** *Every finite metric space D admits a unique coherent decomposition $D = D_1 + \cdots + D_k + D'$, where $D_1, \ldots, D_k$ are linearly independent split metrics and D' is a split-prime metric.*

This theorem is useful for evolutionary biology because it offers a polyhedral framework for phylogenetic reconstruction. Suppose we are given n taxa, for instance the genomes of n organisms, and we take D be a matrix of distances among these taxa. In typical applications, d_{ij} would be the Jukes-Cantor distance [21, §4.4] derived from a pairwise alignment of genome i and genome j. Then we consider the polyhedral complex $\mathrm{Bd}(P_D)$ whose cells are the bounded faces of the polyhedron P_D. This is a contractible complex known as the *tight span* [9] of the metric space D. The metric D is a *tree metric* if and only if the tight span $\mathrm{Bd}(P_D)$ is one-dimensional, and, in this case, the one-dimensional contractible complex $\mathrm{Bd}(P_D)$ is precisely the *phylogenetic tree* which represents the metric D.

The space of phylogenetic trees on n taxa was introduced by Billera, Holmes and Vogtmann [4]. Since every tree metric uniquely determines its tree, this space is a subset of the metric cone. It can be characterized as follows:

Corollary. *The space of trees of* [4] *equals the following subset of the metric cone:*

$$\mathrm{Trees}_n \quad = \quad \bigl\{ D \in \mathbb{R}^{\binom{n}{2}} \, : \, D \textit{ is a metric } \; \textit{and} \; \; \textit{dim}\, \mathrm{Bd}(P_D) \, \leq \, 1 \bigr\}.$$

If the metric D arises from real data then it is unlikely to lie exactly in the space of trees. Standard methods used by biologists, such as the neighbor joining algorithm, compute a suitable projection of D onto Trees_n. From a mathematical point of view, however, it is desirable to replace the concept of a tree by a higher-dimensional

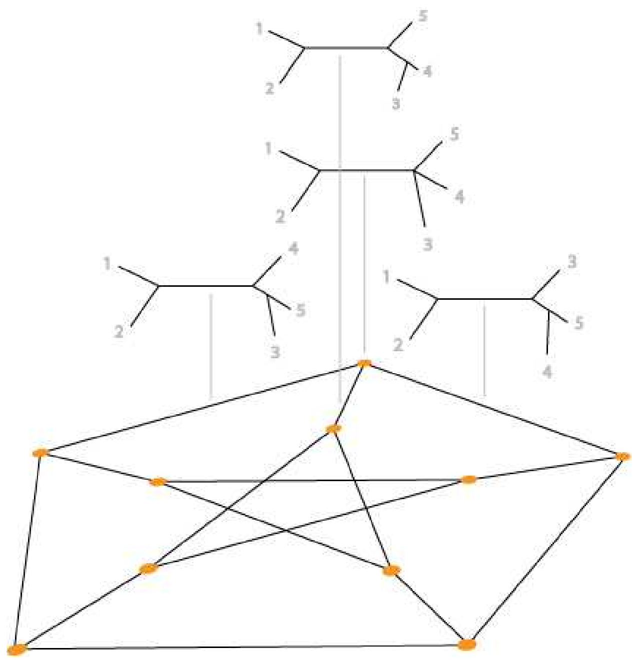


FIGURE 1. The space of phylogenetic trees on five taxa is a seven-dimensional polyhedral fan inside the ten-dimensional metric cone. It has the combinatorial structure of the Petersen graph, depicted here. The fan Trees_5 consists of 15 maximal cones, one for each edge of the graph, which represent the trivalent trees. They meet along 10 six-dimensional cones, one for each vertex of the graph.

object that faithfully represents the data. The tight span $\text{Bd}(P_D)$ is the universal object of this kind. It can be computed using the software `POLYMAKE`. Figure 2 shows the tight span of a metric on six taxa. This metric was derived from an alignment of DNA sequences of six bees. For details and an introduction to `POLYMAKE` we refer to [14]. We note that, for larger data sets, the tight span is often too big. This is where Theorem 1 enters the scene: what one does is remove the *splits residue* D' from the data D. The remaining split-decomposable metric $D_1 + \cdots + D_k$ can be computed efficiently with the software `SPLITSTREE` due to Huson and Bryant [15]. It is represented by a *phylogenetic network*.

Andreas Dress now serves as director of the Institute for Computational Biology in Shanghai (`www.icb.ac.cn`), a joint Chinese-German venture. He presented his theory at the November 2005 workshop at the Clay Mathematics Institute in Cambridge. In his invited lecture at the 1998 ICM in Zürich, Dress suggested that the *"the tree of life is an affine building"* [10]. Affine buildings are highly symmetric infinite simplicial complexes which play an important role in several areas of mathematics, including group theory, representation theory, topology and harmonic analysis.

The insight that phylogenetic trees, and possible higher-dimensional generalizations thereof, are intimately related to affine buildings is an important one. The author of this article agrees enthusiastically with Dress' point of view, as it is consistent with recent advances at the interface of phylogenetics and tropical geometry. An interpretation of tree space as a Grassmannian in tropical algebraic geometry was given in [24]: Figure 1 really depicts a Grassmannian together with its tautological vector bundle. It is within this circle of ideas that the next theorem was found, three years ago, by Lior Pachter and Clay Research Fellow David Speyer [20].

Let T be a phylogenetic tree with leaves labeled by $[n] = \{1, 2, \ldots, n\}$, and with a non-negative length associated to each edge of T. Then we define a real-valued function $\delta^{T,m}$ on the m-element subsets I of $[n]$ as

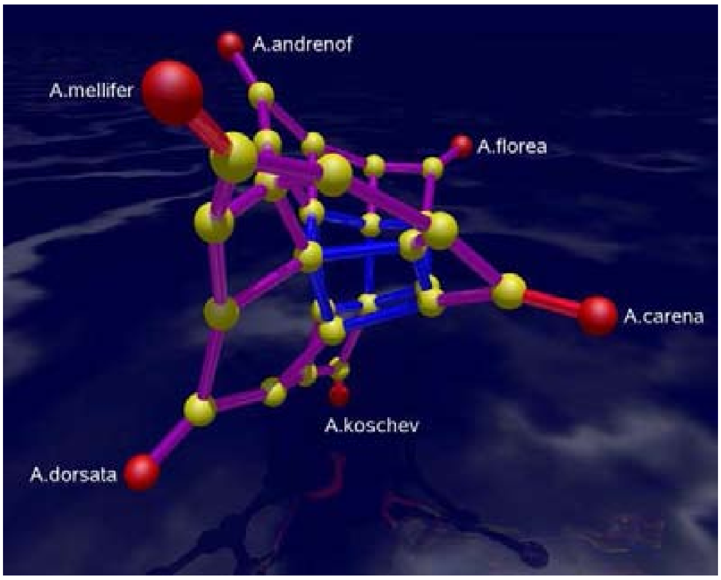


FIGURE 2. The tight span of a six-point metric space derived from aligned DNA sequences of six species of bees. We thank Michael Joswig and Thilo Schröder for drawing this diagram and allowing us to include it. See [14] for a detailed description.

follows: the number $\delta^{T,m}(I)$ is the sum of the lengths of all edges in the subtree spanned by I. For $m = 2$ we recover the tree metric $D_T = \delta^{T,2}$. We call $\delta^{T,m} : \binom{[n]}{m} \to \mathbb{R}$ the *subtree weight function.*

Theorem 2. (Pachter-Speyer Reconstruction from Subtree Weights [20]**)** *Suppose that $n \geq 2m - 1$. Every phylogenetic tree on n taxa is uniquely determined by its subtree weight function. More precisely, $\delta^{T,m}$ determines the tree metric $\delta^{T,2}$.*

The punchline of this theorem is a statistical one. The aim of replacing $m = 2$ by larger values of m is that $\delta^{T,m}$ can be estimated from data in a more reliable manner. Practical advantages of this method were shown in [19].

Phylogenetics has spawned several different research directions in current mathematics, especially in combinatorics and probability. For more information, we recommend the book by Semple and Steel [23], and the special semester on Phylogenetics which will take place in Fall 2007 at the Newton Institute in Cambridge, England.

Algebraists, geometers and topologists may also enjoy a glimpse of *phylogenetic algebraic geometry* [13]. Here the idea is that statistical models of biological sequence evolution can be interpreted as algebraic varieties in spaces of tensors. This approach has led to a range of recent developments which are of interest to algebraists; see [1, 18, 25] and the references given there. As an illustration, we present a recent theorem due to Buczynska and Wisniewski [5]. The abstract of their preprint leaves no doubt that this is an unusual paper as far as mathematical biology goes: *"We investigate projective varieties which are geometric models of binary symmetric phylogenetic 3-valent trees. We prove that these varieties have Gorenstein terminal singularities (with small resolution) and they are Fano varieties of index 4...."*.

The varieties studied here are all embedded in the projective space $\mathbb{P}^{2^{n-1}-1} = \mathbb{P}(\mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \cdots \otimes \mathbb{C}^2)$ whose coordinates x_I are indexed by subsets I of $\{1, \ldots, n\}$ whose cardinality $|I|$ is even. We fix a trivalent tree T whose leaves are labeled by $1, \ldots, n$. Each of the $2n - 3$ edges e of the tree T is identified with a projective line $\mathbb{P}^1$ with homogeneous coordinates $(u_e : v_e)$. For any even subset I of the leaves of T there exists a unique set Paths(I) of disjoint paths, consisting of edges of T, whose end points are the leaves in I. This observation gives rise to a birational morphism

$$\phi_T \,:\, (\mathbb{P}^1)^{2n-3} \to \mathbb{P}^{2^{n-1}-1} \quad \text{defined by} \quad x_I \;=\; \prod_{e\in \text{Paths}(I)} u_e \cdot \prod_{e\notin \text{Paths}(I)} v_e .$$

The closure of the image of ϕ_T is a projective toric variety which we denote by X_T.

Theorem 3. (Buczynska-Wisniewski Flat Family of Trees [5]**)** *All toric varieties X_T are the same connected component of the Hilbert scheme of projective schemes, as T ranges over all combinatorial types of trivalent trees with $n+1$ leaves. Combinatorially, this means that the convex polytopes associated with these toric varieties all share the same Ehrhart polynomial (a formula for this Ehrhart polynomial is given in* [5, §3.4]*).*

Earlier work with Seth Sullivant [25] had shown that the homogeneous prime ideal of X_T has a Gröbner basis consisting of quadrics. These quadrics are the 2×2-minors of a collection of matrices, two for each edge e of T. After relabeling we may assume that the edge e separates the leaves $1, 2, \ldots, i$ from the leaves $i+1, \ldots, n$. We construct two matrices M^e_{even} and M^e_{odd} each having 2^{i-1} rows and 2^{n-i-1} columns. The rows of M^e_{even} are indexed by subsets $I \subset \{1, \ldots, i\}$ with $|I|$ even and the columns are indexed by subsets $J \subset \{i+1, \ldots, n\}$ with $|J|$ even. The entry of M^e_{even} in row I and column J is the unknown $x_{I\cup J}$. The matrix M^e_{odd} is defined similarly. Our Gröbner basis for the toric variety X_T consists of the 2×2-minors of the matrices M^e_{even} and M^e_{odd} where e runs over all $2n - 3$ edges of the tree T. In light of Theorem 3, it would be interesting to decide whether all the X_T lie on the same irreducible component of the Hilbert scheme, and, if yes, to explore possible connections between the generic point on that component to the quadratic equations derived by Keel and Tevelev [17] for the moduli space $\bar{M}_{0,n}$.

The toric variety X_T is known to evolutionary biologists as the *Jukes-Cantor model*. For some applications, it is more natural to study the *general Markov model*. This is a non-toric projective variety in tensor product space which generalizes secant varieties of Segre varieties [18]. The state of the art on the algebraic geometry of these models appears in the work of Elizabeth Allman and John Rhodes [1].

For our last theorem, we leave the field of phylogenetics and turn to mathematical developments inspired by other problems in biological sequence analysis. These problems include *gene prediction*, which seeks to identify genes inside genomes, and *alignment*, which aims to find the biological relationships between two genomes. See [22, §4] for an introduction aimed at mathematicians. Current algorithms for *ab initio* gene prediction and alignment are based on methods from statistical learning theory, and they involve *hidden Markov models* and more general *graphical models*.

From the perspective of algebraic statistics [21], a graphical model is a highly structured polynomial map from a low-dimensional space of parameters to a tensor product space, like the $\mathbb{P}^{2^{n-1}-1}$ we encountered in Theorem 3. It is from this algebraic representation of graphical models that the following theorem was derived:

Theorem 4. (Elizalde-Woods' Few Inference Functions) [11, 12]**)** *Consider a graphical model G with d parameters, where d is fixed, and let E be the number of edges of G. Then the number of inference functions of the model is at most $O(E^{d(d-1)})$.*

We need to explain what an inference function is and what this theorem means. A graphical model is given by a polynomial map $p : \mathbb{R}^d \to \mathbb{R}^N$ where d is fixed and each coordinate p_i is a polynomial of degree $O(E)$ in d unknown parameters. The polynomial p_i represents the probability of making the i-th observation $\#i$, out of a total of N possible observations. The number N is allowed to grow, and in biological applications it can be very large, for instance $N = 4^{1,000,000}$, the number of DNA sequences with one million base pairs.

The monomials in p_i correspond to the possible *explanations* of this observation, where the monomial of largest numerical value will be the most likely explanation. Let Exp be the set of all possible explanations for all the N observations. For a fixed generic choice of parameters $\theta \in \mathbb{R}^d$, we obtain a well-defined function

$$\phi_\theta \,:\, \{1, 2, \ldots, N\} \to \text{Exp}$$

which assigns to each observation its most likely explanation. Any such function, as θ ranges over (a suitable open subset of) $\mathbb{R}^d$ is called an *inference function* for the model f. The number $|\text{Exp}|^N$ of all conceivable

Feature article

functions is astronomical. The result by Elizalde and Woods says that only a tiny, tiny fraction of all these functions are actual inference functions. The polynomial growth rate in Theorem 4 makes it feasible, at least in principle, to pre-compute all such inference functions ahead of time, once per graphical model. This is important for *parametric inference*. Two recent examples of concrete bio-medical applications of parametric inference can be found in [3] and [7]. One way you can tell a biology paper from a mathematics paper is that the order of the authors' names has a meaning and is thus rarely alphabetic.

This concludes my discussion of four recent theorems that were inspired by biology. All four stem from my own limited field of expertise, and hence the selection has been very biased. A feature that Theorems 1, 2, 3 and 4 have in common is that they are meaningful as statements of pure mathematics. I must sincerely apologize to my colleagues in mathematical biology for having failed to give proper credit to their many many important research contributions. My only excuse is the hope that they will agree with my view that the answer to the question in the title is affirmative.

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Selected Articles by Research Fellows

DANIEL BISS

"Large annihilators in Cayley-Dickson algebras," with Daniel Dugger and Daniel C. Isaksen www.front.math.ucdavis.edu/math.RA/0511691–250

"On the zero-divisors in the 32-dimensional Cayley-Dickson algebra."

MANJUL BHARGAVA

"Higher composition laws III: The parametrization of quartic rings," Ann. of Math., vol. 159, no. 3 (2004), 1329–1360.

"The density of discriminants of quartic rings and fields," Ann. of Math. 162 (2005), no. 2, 1031–1063.

ALEXEI BORODIN

"Markov processes on partitions, probability theory and related fields," with G. Olshanski, vol. 135 (2006), 84–152.

"Averages of characteristic polynomials in random matrix theory," with E. Strahov, Communications on Pure and Applied Mathematics, vol. LIX (2006), 161–253.

MARIA CHUDNOVSKY

"The Strong Perfect Graph Theorem," with N. Robertson, P. Seymour and R. Thomas, to appear in the Ann. of Math.

"The structure of Clawfree Graphs," with Paul Seymour, Surveys in Combinatorics 2005, London Math Society Lecture Note Series, vol. 327.

BEN GREEN

"An inverse theorem for the Gowers U3 norm," with Terence Tao, www.arxiv.org/math.NT/0503014

"Freiman's theorem in an arbitrary abelian group," with Imre Ruzsa, to appear in Journal of London Math. Soc., www.arxiv.org/math.NT/0505198

SERGEI GUKOV

"The superpolynomial for knot homologies," with N. M. Dunfield and J. Rasmussen, to appear in the Experiment. Math., www.arxiv.org/abs/math.GT/0505662

"Matrix factorizations and Kauffman homology," joint with J. Walcher, www.arxiv.org/abs/hep-th/0512298

BO'AZ KLARTAG

"Fitting a Cm-Smoenkatesh," with C. Fefferman, submitted, 24 pages.

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ELON LINDENSTRAUSS

"Invariant measures and arithmetic quantum unique ergodicity," Ann. of Math. (2) 163 (2006), no. 1, 165–219.

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CIPRIAN MANOLESCU

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MARYAM MIRZAKHANI

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DAVID SPEYER

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"Computing Tropical Varieties," www.arxiv.org/abs/math.AG/0507563

ANDRÁS VASY

"Scattering theory on SL(3)/SO(3): Connections with quantum 3-body scattering," with R. Mazzeo, to appear in Proc. London Math. Soc.

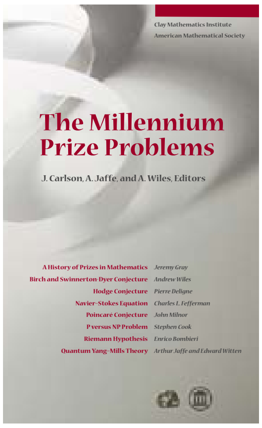
"Geometric optics and the wave equation on manifolds with corners," to appear in the Contemporary Mathematics series of the AMS, volume "Recent Advances in Differential Equations and Mathematical Physics."

AKSHAY VENKATESH

"Local-global principles for representations of quadratic forms," with Jordan Ellenberg, arxiv.org/math.NT/0604232

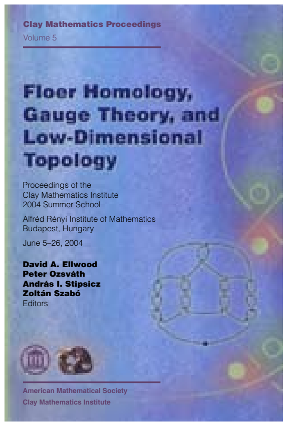
"Equidistribution, L-functions and ergodic theory: On some problems of Yu. V. Linnik," with Philippe Michel, to appear in ICM 2006 proceedings.

Books & Videos



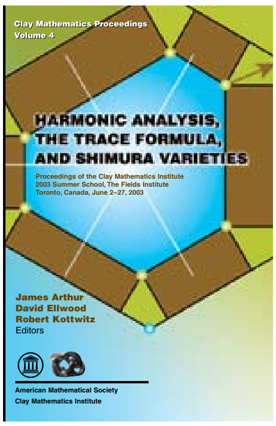
The Millennium Prize Problems; This volume gives the official description of each of the seven problems as well as the rules governing the prizes. It also contains an essay by Jeremy Gray on the history of prize problems in mathematics. *See page 3.*

Floer Homology, Gauge Theory, and Low-Dimensional Topology; Proceedings of the 2004 CMI Summer School at Rényi Institute of Mathematics, Budapest. Editors: David Ellwood, Peter Ozsváth, András Stipsicz, and Zoltán Szábo. This volume grew out of the summer school that took place in June of 2004 at the Alfréd Rényi Institute of Mathematics in Budapest, Hungary. It provides a state-of-the-art introduction to current research, covering material from Heegaard Floer homology, contact geometry, smooth four-manifold topology, and symplectic four-manifolds.



Lecture Notes on Motivic Cohomology; Authors: Carlo Mazza, Vladimir Voevodsky, Charles Weibel. This book provides an account of the triangulated theory of motives. Its purpose is to introduce the reader to Motivic Cohomology, develop its main properties and finally to relate it to other known invariants of algebraic varieties and rings such as Milnor K-theory, étale cohomology and Chow groups.

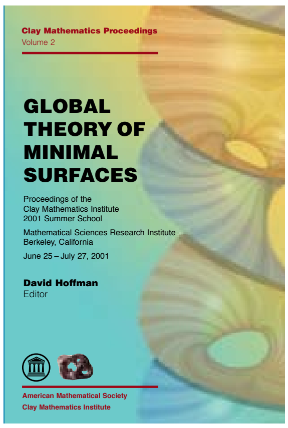
Surveys in Noncommutative Geometry; Editors: Nigel Higson, John Roe. In June 2000 a summer school on Noncommutative Geometry, organized jointly by the American Mathematical Society and the Clay Mathematics Institute, was held at Mount Holyoke College in Massachusetts. The meeting centered around several series of expository lectures which were intended to introduce key topics in noncommutative geometry to mathematicians unfamiliar with the subject. Those expository lectures have been edited and are reproduced in this volume.



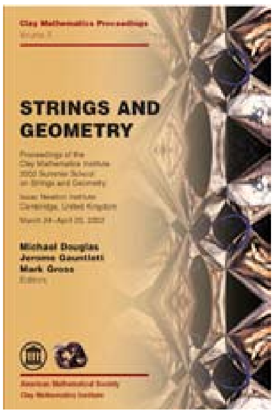
Harmonic Analysis, the Trace Formula and Shimura Varieties; Proceedings of the 2003 CMI Summer School at Fields Institute, Toronto. Editors: James Arthur, David Ellwood, Robert Kottwitz. CMI/AMS, 2005, 689 pp. www.claymath.org/publications/Harmonic_Analysis

The subject of this volume is the trace formula and Shimura varieties. These areas have been especially difficult to learn because of a lack of expository material. This volume aims to rectify that problem. It is based on the courses given at the 2003 Clay Mathematics Institute Summer School. Many of the articles have been expanded into comprehensive introductions, either to the trace formula or the theory of Shimura varieties, or to some aspect of the interplay and application of the two areas.

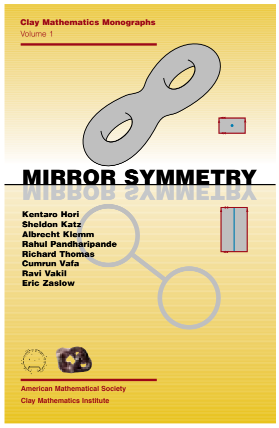
Global Theory of Minimal Surfaces. Proceedings of the 2001 CMI Summer School at MSRI. Editor: David Hoffman. CMI/AMS, 2005, 800 pp. www.claymath.org/publications/Minimal_Surfaces



This book is the product of the 2001 CMI Summer School held at MSRI. The subjects covered include minimal and constant-mean-curvature submanifolds, geometric measure theory and the double-bubble conjecture, Lagrangian geometry, numerical simulation of geometric phenomena, applications of mean curvature to general relativity and Riemannian geometry, the isoperimetric problem, the geometry of fully nonlinear elliptic equations and applications to the topology of three-manifolds.



Strings and Geometry; Proceedings of the 2002 CMI Summer School held at the Isaac Newton Institute for Mathematical Sciences, UK. Editors: Michael Douglas, Jerome Gauntlett and Mark Gross. CMI/AMS publication, 376 pp., Paperback, ISBN 0-8218-3715-X. List: $69. AMS Member: $55. Order code: CMIP/3. To order, visit www.ams.org/bookstore



Mirror Symmetry. Authors: Kentaro Hori, Sheldon Katz, Albrecht Klemm, Rahul Pandharipande, Richard Thomas, Ravi Vakil. Editors: Cumrun Vafa, Eric Zaslow. CMI/AMS publication, 929 pp., Hardcover. ISBN 0-8218-2955-6. List: $124. AMS Members: $99. CMIM/1. To order, visit www.ams.org/bookstore

Strings 2001. Authors: Atish Dabholkar, Sunil Mukhi, Spenta R. Wadia. Tata Institute of Fundamental Research. Editor: American Mathematical Society (AMS), 2002, 489 pp., Paperback,ISBN0-8218-2981-5,List$74.,AMSmembers:$59. Order code: CMIP/1. To order, visit www.ams.org/bookstore

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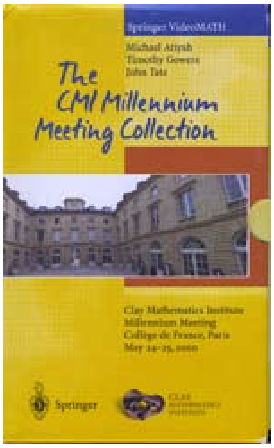

The CMI Millennium Meeting Collection. Authors: Michael Atiyah, Timothy Gowers, John Tate, François Tisseyre. Editors: Tom Apostol, Jean-Pierre Bourguignon, Michele Emmer, Hans-Christian Hege, Konrad Polthier. Springer VideoMATH, © Clay Mathematics Institute, 2002. Box set consists of four video cassettes: The CMI Millennium Meeting, a film by François Tisseyre; The Importance of Mathematics, a lecture by Timothy Gowers; The Millennium Prize Problems, a lecture by Michael Atiyah; and The Millennium Prize Problems, a lecture by John Tate. VHS/NTSC or PAL. ISBN 3-540-92657-7, List: $119, EUR 104.95. To order, visit www.springer-ny.com (in the United States) or www.springer.de (in Europe)

These videos document the Paris meeting at the Collège de France where CMI announced the Millennium Prize Problems. For anyone who wants to learn more about these seven grand challenges in mathematics.

Videos of the 2000 Millennium event are available online and in VHS format from Springer-Verlag. To order the box set or individual tapes visit www.springer.com

Profile of Bow Street Staff

Standing from left to right: Vida Salahi, Candace Bott, Eric Woodbury, Elizabeth Abraham, Zareh Orchanian, and James Carlson. Seated: Christa Carter and David Ellwood

James A Carlson

As president of CMI, Jim is responsible for the overall scientific direction and administration of the Institute, consultation with the scientific advisory board, liaison with the mathematical community, selection of and liaison with the Institute's fellows and scholars, and development of initiatives and nominations for consideration by the board.

Jim grew up in Idaho and received BS and PhD degrees in mathematics from the University of Idaho and Princeton University in 1964 and 1971, respectively. In 1975, after postdoctoral positions at Stanford and Brandeis, he moved to the University of Utah. Since 2003 he has served as president of CMI. Jim's research area is algebraic geometry.

Eric G Woodbury

As Chief Administrative Officer, Eric is responsible for the nonscientific affairs of the Institute, including legal and accounting matters (such as Board and IRS reporting), personnel issues and investment of the endowment.

Eric has worked with Landon Clay as a business lawyer for many years, after earning BA, JD and MEd degrees from Boston College. Eric has two children and spends vacation time on Swan's Island, Maine.

David A Ellwood

David Alexandre Ellwood has been the Research Director of CMI since November 2004. He was Resident Mathematician at CMI from 1999 to 2004 and has held positions at Harvard University, Boston University, Strasbourg University, ETH (Zürich), Université de Paris VI and the Institute des Hautes Etudes Scientifiques. His research has centered around ideas related to Connes' theory of non-commutative geometry, operator algebras, and, more recently, the theory of operads and its applications. His other interests include theoretical physics and the foundations of mathematics.

As Research Director of CMI, David directs the Liftoff program, CMI's annual Summer Schools, the Clay Research Academy, CMI's workshops and conferences, oversees CMI research fellows and scholars, and is acting editor of CMI's publications series. David also founded CMI's web archive project, which seeks to provide online access to the collected works of leading mathematicians. David has served on the organizing committee of more than 50 international conferences and workshops and is an editor of "Harmonic Analysis, the Trace Formula, and Shimura Varieties" together with James Arthur and Robert Kottowitz.

Vida Salahi

As Publications, Web and Program Manager, Vida is responsible for development, editing, and production of CMI publications, development of the CMI website, and management of the Research Academy and other high school programs. Vida is also CMI's Research Operations Analyst.

Vida grew up in Tehran, Iran, where, after earning a BA in mathematics at Shahid Beheshti University, she taught mathematics for several years. She joined CMI in 2001.

Candace C Bott

As Media and Public Relations Manager, Candace is responsible for developing and coordinating printed program materials, including CMI's Annual Report. She is also in charge of media relations, public lectures and events planning, and oversees CMI's video and photographic materials.

Candace holds a PhD in Art History from the University of Illinois and has studied and worked abroad in Italy, France, and Spain. Before joining CMI in 2000, she developed television documentaries for WGBH Boston. She is the daughter of the late Hungarian topologist Professor Raoul Bott of Harvard. Candace lives in Cambridge with her husband and four-year-old daughter.

Elizabeth A C Abraham

As Executive and Program Assistant Elizabeth is responsible for liaison and other support to the President and Research Director on scientific programs, and management of Bow Street workshops.

Elizabeth graduated from Scripps College in Claremont, California, in 1998 with a degree in English. She is currently pursuing a graduate certificate in Forensic Accounting. She lives in Dorchester with her husband.

Christa D Carter

As Office Manager, Christa is responsible for day-to-day office administration, video and poster order fulfillment, maintenance of facilities, and assistance with Board of Director meetings.

Christa joined CMI in 2006; she graduated from Gordon College in 2005 with a B.A. in Medieval History. She has lived in Germany, Russia and Nicaragua, and speaks Spanish and German.

Zareh M Orchanian

As CMI's Accountant, Zareh is responsible for bookkeeping and financial reporting.

"Z" is a CPA. He has worked in the accounting field for over 20 years. He graduated from Bentley College with a degree in accounting. He has two children, and lives in Waltham, Mass.

2006 Institute Calendar

JANUARY FEBRUARY	Senior Scholar: Yongbin Ruan, MSRI Program on New Topological Methods in Physics. January 9–May 19 Senior Scholar: Jean-Louis Colliot-Thélène, Rational and Integral Points on Higher-Dimensional Varieties, Mathematical Sciences Research Institute. January 9–May 19 Eigenvarieties Conference, Harvard University, Cambridge, MA. February 1–May 31 Lie Groups, Dynamics, Rigidity and Arithmetic, a conference in honor of Gregory Margulis, Yale University, New Haven, CT. February 24–27
MARCH APRIL MAY	Additive Combinatorics Conference, Centre de Recherche Mathématiques, Montreal. March 30–April 5 Conference on Automorphic Forms and L-Functions, Weizmann Institute of Science, Tel Aviv. April 14–16 Clay Public Lecture by Persi Diaconis: Mathematics and Magic Tricks, Ray and Maria Stata Center at MIT, Kirsch Auditorium. April 25 Eigenvarieties Weekend Workshop, CMI, Cambridge, MA. May 10–15 International Conference on Global Dynamics Beyond Uniform Hyperbolicity, Northwestern University. May 17–22
JUNE JULY AUGUST	Conference on Hodge Theory, Venice International University. June 19–24 Affine and Double Affine Hecke Algebras & The Langlands Program Workshops, Centre International de Rencontres Mathématiques, Luminy, France. June 19–July 14 Senior Scholars: Yakov Eliashberg, Robion Kirby and John Milnor, Park City Mathematical Institute Program on Low Dimensional Topology. July 1–30 CMI Summer School on Arithmetic Geometry, Goettingen, Germany. July 17–August 11
SEPTEMBER OCTOBER NOVEMBER DECEMBER	Clay Mathematics Institute Workshop on Moduli spaces of Vector Bundles, Tufts–Boston University, Senior Scholar Peter Newstead, October Clay Public Lecture on the **P** vs **NP** Problem by Michael Sipser. Harvard Science Center. October 17 CMI Annual Meeting, Columbia University, November Workshop on Stochastic Geometry and Field Theory, KITP, Santa Barbara, CA, August 1–December 31 Moduli Spaces and Vector Bundles (CIMAT, Mexico), December 11–15