Final Exam

Math 1B Section 3

Friday August 14, 2009

Name: _____

Problem	Points	Score
1	8	
2	8	
3	14	
4	14	
5	14	
6	14	
7	14	
8	14	
Total	100	

1. You must show all work and justify all answers. Correct answers without sufficient explanation will be worth 0 points. Also, please write legibly.

2. Do not open this exam until told to do so. You will have until 12 PM to complete the exam.

3. Use of books or a calculator is not allowed. You may use one $8\frac{1}{2} \times 11$ sheet of paper with writing on one side as notes.

4. If you need more space for a solution to a problem, use the back of the test sheets—but clearly indicate which problem you are solving and a path to your complete solution. Do not separate the pages of the exam.

5. Read directions carefully and do all that is asked.

1. (8 points) Show that $y = x - x^{-1}$ is a solution to the initial value problem

$$xy' + y = 2x \qquad\qquad y(1) = 0$$

2. (8 points) Find the general solution to

$$x + yy'e^{-x} = 0$$

3. (14 points) A tank contains 10 L of pure water. A solution with a salt concentration of 1 kg/L is added at a rate of 4 L/min. The solution is kept mixed and is drained from the tank at a rate of 2 L/min. The tank can hold a maximum of 20 L. Find the amount of salt in the tank at the moment when it begins to overflow.

4. (14 points) Find the general solution to $y'' + y = \sec x \qquad \frac{-\pi}{2} < x < \frac{\pi}{2}$.

5. (14 points) Use power series to solve the initial value problem

$$y'' + xy' + y = 0 \qquad\qquad y(0) = 0 \qquad y'(0) = 1$$

6. (14 points) Determine the form of y_p for the method of undetermined coefficients for the following differential equations. You do NOT need to determine the coeffients.

(a) $y'' + 3y' = 2 + xe^{-3x} + \sin x$

(b) $y'' - 5y' + 6y = e^x \cos 2x + (3x + 4)e^{2x} \sin x$

7. (14 points) Suppose a spring has mass m and spring constant k and let $\omega = \sqrt{k/m}$. Suppose that there is no damping force, but the mass is being driven by a force $F(t) = F_0 \sin(\omega_0 t)$, where $\omega_0 \neq \omega$. (F_0, ω, and ω_0 are all constants.) Find a formula for the motion of the mass as a function of time. (You will need to write down a differential equation describing this system and find its general solution.)

8. (14 points) Consider the following differential equation $\frac{dy}{dx} = e^y(y-1)^2(y-4)$. You will not be solving this differential equation.

(a) Find the equilibrium solutions to this differential equation.

(b) Assuming $y(0) = 3$, without solving the differential equation, describe what happens to y as $t \to \infty$.