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MATH 140 MIDTERM #1

Do not write your answers on this sheet. Instead please write your name, your student ID, and all your answers in your blue books. Total: 100 pts., 50 minutes.

(1) (5 pts. each) For each of (a)-(d) below: If the proposition is true, write TRUE. If the proposition is false, write FALSE. (Please do not use the abbreviations T and F, since in handwriting they are sometimes indistiguishable.) No explanations are required in this problem.

(a) The set $\{(x, |x|, 0) : x \in \mathbb{R}\}$ is the trace of a parametrized differentiable curve in $\mathbb{R}^3$.

(b) If S is a regular surface, and $a \in S$, then there is an open neighborhood V of a in $\mathbb{R}^3$ such that $V \cap S$ is a regular surface diffeomorphic to some open subset of $\mathbb{R}^2$.

(c) If S_1 and S_2 are regular surfaces, and there is a differentiable map $\phi\colon S_1 \to S_2$ whose derivative $d\phi_p\colon T_p(S_1) \to T_{\phi(p)}(S_2)$ is invertible at each $p \in S_1$, then S_1 and S_2 are diffeomorphic.

(d) There exists exactly one regular parametrized curve $\alpha : \mathbb{R} \to \mathbb{R}^3$ satisfying all of the following conditions:

- α is parametrized by arc length.
- $\alpha(0) = (0, 0, 0)$.
- α has curvature $k(s) = s^2 + 1$ at all $s \in \mathbb{R}$.
- α has torsion $\tau(s) = 3s$ at all $s \in \mathbb{R}$.

(2) (20 pts.) Let L be the line in $\mathbb{R}^3$ passing through $(3, 2, 1)$ and $(2, 1, 3)$. Let P be the plane in $\mathbb{R}^3$ passing through $(0, 0, 1)$, $(1, 1, 2)$, and $(2, 3, 5)$. Find the angle formed by L and P. (Your answer should be between 0 and $\pi/2$.)

(3) (20 pts.) Define a parametrized curve by

$$\alpha(s) = \left(7 + \frac{3s}{5}, 4\sin\frac{s}{5}, 4\cos\frac{s}{5}\right)$$

for $s \in \mathbb{R}$. You may assume that α is parameterized by arc length. (It is.) Calculate the radius of curvature $R(s)$ of α at s, as a function of s. Show your work.

(4) (40 pts.) For which real numbers c is the set of (x, y, z) satisfying

$$x^2 + y^2 + \sin z = c$$

a regular surface? (Don't forget to explain why it's not regular in the cases that you say it's not!)

This is the end! At this point, you may want to look over this sheet to make sure you have not omitted any problems. Check that your answers make sense! Please take this sheet with you as you leave.