Math 121B Final 2004 May 18 8:00-11:00am .   R. Borcherds

Please make sure that your name is on everything you hand in.
You are allowed calculators and 1 sheet of notes.
All questions have about the same number of marks.

1. Evaluate
$$\int_0^\infty x^{10} e^{-x^2}\, dx.$$
   (Hint: put $y = x^2$.)

2. If $|x| = \sum_l c_l P_l(x)$ for $|x| \le 1$ is the expansion of $|x|$ as a Legendre series, then find the coefficients c_l for $l = 0, 1, 2$.

3. Evaluate
$$\lim_{n \to \infty} \frac{\Gamma(n + 1/2)}{\sqrt{n}\,\Gamma(n)}.$$

4. By repeated integration by parts, find the first 3 terms of the asymptotic series for
$$\int_x^\infty t^{n-1} e^{-t}\, dt.$$

5. Find a nonzero solution of the following differential equation:
$$x(x-1)^2 y'' = 2y.$$

6. A vibrating string of length π whose displacement y satisfies the equation
$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2}$$
   has the initial conditions $y(x, 0) = 2\sin(x)\cos(x)$ and has zero initial velocity. Find $y(x, t)$ for all x ($0 \le x \le \pi$) and all $t \ge 0$.

7. What is the probability that a random integer n, $1 \le n \le 999$, is divisible by both 6 and 10? What is the probability that it is divisible by 6 given that it is divisible by 10?

8. In a family of 3 children, what is the probability that all three are girls given that at least two are girls? What is the probability that all three are girls given that the oldest two are girls?

9. How many solutions are there to the equation

$$a + b + c + d = 20$$

where a, b, c, d are non-negative integers?

10. If there are an average of two misprints per page of a long book, what is the chance that a given page has no misprints? What is the chance that a given page has at least two misprints?