Math 115 Professor K. A. Ribet
Second Midterm Exam October 28, 1999

This is a closed-book exam: no notes, books or calculators are allowed. Explain your answers in complete English sentences. No credit will be given for a "correct answer" that is not explained fully. Don't worry too much about simplifying arithmetical expressions; "$3 \cdot 5 + 1$" is the same answer as "16" in most contexts.

**1** *(5 points)*. Suppose that $n$ and $m$ are positive integers, that $p$ is a prime and that $\alpha$ is a non-negative integer. Assume that $n$ is divisible by $p^\alpha$, that $m$ is prime to $p$ and that $F = \dfrac{n}{m}$ is an integer. Show that $F$ is divisible by $p^\alpha$.

**2** *(6 points)*. Let $f(x)$ be a polynomial with integer coefficients that satisfies $f(1) = f'(1) = 3$. Calculate the remainder when $f(-18)$ is divided by $19^2$.

**3** *(5 points)*. Determine the number of solutions to the congruence $x^2+x+1 \equiv 0 \mod 7^{11}$.

**4** *(6 points)*. Find an integer $n \geq 1$ so that $a^{3n} \equiv a \mod 85$ for all integers $a$ that are divisible neither by 5 nor by 17.

**5** *(6 points)*. Find the number of solutions mod 120 to the system of congruences
$$x \equiv \begin{cases} 2 & \mod 4 \\ 3 & \mod 5 \\ 4 & \mod 6 \end{cases}.$$

**6** *(7 points)*. If $m = 15709$, we have $2^{(m-1)/2} \equiv 1 \mod m$ and $2^{(m-1)/4} \equiv 2048 \mod m$. With the aid of these congruences, one can find quite easily a positive divisor of $m$ that is neither 1 nor $m$. Explain concisely: how to find such a divisor, and why your method works.