George M. Bergman | Fall 1998, Math 1AW | 13 November, 1998

891 Evans | **Second Midterm – make-up exam** | 3:10-4:00 PM

1. (50 points, 10 points apiece) Find the following.

(a) $\lim_{x \to 2} \dfrac{\sinh x - \sinh 2}{\sinh 2x - \sinh 4}$.

(b) $\lim_{x \to 0} \dfrac{\ln \cos 5x}{x^2}$.

(c) An antiderivative of the function $1 + e^x + 2 \sin x$

(d) A function satisfying the differential equation $f'(x) = 2 f(x)$, and such that $f(1) + f'(1) + f''(1) = 1$.

(e) $\dfrac{d}{dx} \sin (x^{\cos x})$

2. (25 points) (a) (10 points) State the principle of mathematical induction.

(b) (15 points) Suppose f is an infinitely differentiable function (i.e., a function such that $f', f'', \ldots, f^{(n)}, \ldots$ all exist), which satisfies the equation $f'(x) = 2 f(x) + 1$. Prove that for all positive integers n, one has $f^{(n)}(x) = 2^n f(x) + 2^{n-1}$. Here $f^{(n)}$ means the nth derivative of f, and $f^{(0)}$ means f. Suggestion: Use mathematical induction. (Do not try to solve the given equation.)

3. (25 points) (a) (15 points) Give the information asked for below about the curve $y = (\ln x)^2$. If any of the items asked for does not exist, write "None". (For the limit, write "None" only if the function does not approach either a real number or $\pm \infty$.)

(b) (10 points) Sketch the curve. Your sketch does not need to reflect accurate numerical values of the coordinates of the various transition points, as long as it correctly shows the order in which these occur.

domain: _____ x-intercept(s): _____ y-intercept(s): _____ increasing on interval(s): _____ decreasing on interval(s): _____ concave up on interval(s): _____ concave down on interval(s): _____ vertical asymptote(s): _____ $\lim_{x \to \infty} y =$ _____ extrema: _____