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Fall 1998, Math 1AW

First Midterm, Makeup Exam

2 October, 1998
3:10-4:00 PM

1. (32 points, 8 points apiece) Compute the following limits. Give the value if the limit is defined, or if it is ∞ or $-\infty$. If none of these is true, write *No limit.*

(a) $\lim_{x \to -\infty} \dfrac{5x^2 + x^3}{5x^3 - x^2}$.

(b) $\lim_{x \to 0} \dfrac{e^x - 1}{2x}$.

(c) $\lim_{x \to 2^-} \dfrac{x^2 + 4}{\sqrt{x} - \sqrt{4 - x}}$.

(d) $\lim_{x \to \pi} (\sin x)/(x-\pi)$.

2. (36 points, 9 points apiece) Compute the following derivatives. (Note that (c) is a second derivative.)

(a) $\dfrac{d}{dx} \dfrac{1}{x^3 + 2x^2 + 79}$.

(b) $\dfrac{d}{dx} (\sec x^b)^a$, where a and b are real numbers.

(c) $\dfrac{d^2}{dx^2} e^{e^x}$.

(d) $\dfrac{d}{dx} f(x)$, where f is a differentiable function satisfying

$xf(x) - x^2 f(x) - xf(x)^2 = 1$.

3. (12 points) A point q is moving along the parabola $y = x^2$. Express the *rate of change* of its distance from $(0,0)$ at a given moment in terms of x (the x-coordinate of the point) and dx/dt (the rate of change of that coordinate).

4. (a) (8 points) Suppose f is a function and a a real number such that f is differentiable at a. Give the definition of the derivative $f'(a)$.

(b) (12 points) If f is a function and a a real number such that f is differentiable at a, and $f(a) \neq 0$, prove from the above definition a formula for the derivative at a of the function $G(x) = 1/f(x)$ in terms of f and its derivative at a. (You may assume without proof results proved in Stewart about limits; and the result that a differentiable function is continuous, but assume no differentiation formulas. In particular, you may not assume the formula for the derivative of a quotient or for the derivative of a power; though of course you may use either of those formulas in scratch-work to check the formula you get.)