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Fall 1996, Math 53

Second Midterm, make-up version

4 Nov., 1996
10:10-11:00 AM

1. (33 points) (a) (20 points) Find the maximum and minimum values of the function $(x+1)^2+y^2$ on the ellipse $x^2/4 + y^2 = 1$, and say at what points these values occur.

(b) (13 points) Give the gradients $\nabla((x+1)^2+y^2)$ and $\nabla(x^2/4 + y^2)$ at one point of the ellipse where $(x+1)^2+y^2$ assumes its maximum value, and at one point of the ellipse where its assumes it minimum value.

2. (33 points) (a) (13 points) Let D be the region bounded above by the parabola $y = 2x - x^2$ and below by the line $y = 0$. Give the two expressions for the double integral $\int\int xy\, dA$ as an iterated integral (i.e., the expressions using the two orders of integration).

(b) (20 points) Evaluate the above double integral (by any method).

3. (34 points) Suppose f and g are twice-differentiable functions of a real variable (i.e., assume their second derivatives, f'' and g'', exist). Suppose we define a function E of two variables by the equation
(a) (12 points) Express the first and second partial derivatives E_x, E_y, E_{xx}, E_{xy}, E_{yx}, and E_{yy} in terms of f and g.

(b) (6 points) Assume $f(x)$ and $g(x)$ are > 0 for all x. Given a point (x_0, y_0), under what conditions on f and g will (x_0, y_0) be a critical point of the function E?

(c) (16 points) For f and g as in (b), and (x_0, y_0) a critical point of E, find conditions on f and g for this point to be a local maximum, conditions for it to be a local minimum, and conditions for it to be a saddle point. Also state the conditions under which the criteria we have learned *do not determine* the nature of the critical point.