Math 54, Section 1: Differential equations and linear algebra.
Fall 1994, H. W. Lenstra, Jr.
Final examination, December 12, 1994.

Name:

Section number:

T. A.:

List of discussion sections:
101 S. Simic
102 A. Gottlieb
103 G. Anderson
104 G. Anderson
105 S. Simic
106 T. Walker
107 A. Gottlieb
108 L. Pyle
109 L. Pyle

1	
2	
3	
4	
5	
Total	

Problem 1. *(20 points)*
Solve the system of differential equations

$$x_1'(t) = 3x_1(t) + 3x_2(t),$$
$$x_2'(t) = -2x_1(t) - 4x_2(t)$$

with initial conditions

$$x_1(0) = 1, \qquad x_2(0) = 3.$$

Show your work.

Solution:

Problem 2. *(20 points)*

(a) Find three functions $y_1(x)$, $y_2(x)$, $y_3(x)$ defined on $(-\infty, \infty)$ whose Wronskian is given by

$$W(y_1, y_2, y_3)(x) = e^{4x}.$$

(b) Are the functions y_1, y_2, y_3 that you found linearly independent on $(-\infty, \infty)$? Justify your answer.

Solution:

Problem 3. *(20 points)*
Find a homogeneous third-order linear differential equation with constant coefficients that has

$$y(x) = 3 \cdot e^{-x} - \cos(2x)$$

as a solution. Explain how you found it.

What is the general solution of that differential equation?

Solution:

Problem 4. *(20 points)*
For the function $f(x) = e^{x/\pi}$, draw a careful sketch of the graphs of the functions to which the following Fourier series converge on the interval $[0, 4\pi]$:

(a) the Fourier sine series of f on $[0, \pi]$;

(b) the Fourier cosine series of f on $[0, \pi]$;

(c) the ordinary Fourier series of f on $[-\pi, \pi]$.

Pay particular attention to the discontinuities of the functions.

Note: you are *not* asked to compute the coefficients of those Fourier series.

Solution:

Problem 5. *(20 points)*
Find a function $u = u(x,t)$, defined for $0 \leq x \leq \pi$ and $t \geq 0$, satisfying the partial differential equation

$$(*) \qquad u_{xx} = -u_{tt} \qquad (0 < x < \pi, \quad t > 0),$$

with boundary conditions

$$u(0,t) = u(\pi,t) = 0 \qquad (t > 0)$$

and initial conditions

$$u(x,0) = \sin(3x), \quad u_t(x,0) = 0 \qquad (0 < x < \pi).$$

Hint: look for a function of the form $u(x,t) = X(x)T(t)$, as in sec. 10.6 of the textbook; but note that $(*)$ is *not* a wave equation, because of the minus sign.

Solution: