George M. Bergman
155 Dwinelle Hall

Fall 2008, Math 1B
Second Midterm

24 Oct., 2008
3:10-4:00 PM

This is a closed book exam. You are allowed one 2-sided 8½" × 11" sheet of notes.

Attempt all problems. Write solutions on these sheets. Ask for scratch paper if the fronts and backs of these pages are not sufficient; put your name on any such extra sheets and hand them in with your exam.

Credit for an answer may be reduced if a large amount of irrelevant or incoherent material is included along with the correct answer.

Questions begin on the next sheet. Fill in your name and section on this sheet now, but do not turn the page until the signal is given. At the end of the exam, stop writing and close your exam as soon as the ending signal is given, or you will lose points.

Think clearly, stay calm.

Your name ______________________________

Sections: Mark yours with ×.
(Note that they are listed in order of hour, not section-number.)

usual place, hour (MW),		Sec.		TA
171 Stanley	8:00 - 9:00	**201**	☐	Benjamin Tsou
3102 Etcheverry	9:00 - 10:00	**203**	☐	Kiril Datchev
71 Evans	10:00 - 11:00	**204**	☐	Benjamin Tsou
3111 Etcheverry	11:00 - 12:00	**205**	☐	Harold Williams
75 Evans	12:00 - 1:00	**206**	☐	Koushik Pal
70 Evans	1:00 - 2:00	**207**	☐	Gary Sivek
105 Latimer	2:00 - 3:00	**208**	☐	Gary Sivek
3102 Etcheverry	2:00 - 3:00	**211**	☐	Koushik Pal
85 Evans	5:00 - 6:00	**210**	☐	Harold Williams
Other or none			☐	Explain __________

Leave blank for grading

1	/ 48
2	/ 36
3	/ 16
Σ	/100

MUTTS *Patrick McDonnell*

1. (48 points: 12 points each.) Compute the following. If an expression is undefined, say so.

work | *answers:*

(a) $\int_{-1}^{1} x^{-2}\, dx$

(a)

(b) $\lim_{n \to \infty} (2^n + 3^n)/4^n$

(b)

(c) $\Sigma_{n=0}^{\infty} (2^n + 3^n)/4^n$

(c)

(d) The first four terms (i.e., the constant term through the x^3 term) of the Maclaurin series for $(1 + x + x^2)^{-1}$

(d)

2. (36 points, 12 points each) For each of the items listed below, either *give an example* with the property stated, or give a brief reason why *no such example exists.* (If you give an example, you are *not* asked to show that it has the asserted property.)

(a) A power series whose interval of convergence is the open interval $(0, 100)$.

(b) A bounded sequence of real numbers $a_0, a_1, a_2, \ldots,$ which does not converge.

(c) A power series which converges at $x = -1$ but at no other point.

3. (16 points) Suppose $a_0,\ a_1,\ a_2,\ \ldots,\ a_n,\ \ldots$ are real numbers, and that r and s are nonzero real numbers with $|r| < |s|$. Prove that if the sequence $|a_0|,\ |a_1 s|,\ |a_2 s^2|,\ \ldots,$ $|a_n s^n|,\ \ldots$ is bounded, then the series $\Sigma_{n=0}^{\infty} a_n r^n$ converges absolutely.

This was part of a lemma that I proved in class, in proving the theorem on radii of convergence of power series; so of course you cannot quote that lemma, or the theorem on radii of convergence, in proving this. But you can use any of the earlier results we had concerning sums of series.

Though remembering the lecture might help you do this problem, it is not necessary. The proof is a straightforward application of earlier results in the text.

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155 Dwinelle Hall — Solutions to the **Second Midterm** — 3:10-4:00 PM

1. (48 points: 12 points each.) Compute the following. If an expression is undefined, say so.

(a) $\int_{-1}^{1} x^{-2}\,dx$ *Answer: Undefined*

(b) $\lim_{n\to\infty} (2^n+3^n)/4^n$ *Answer:* 0

(c) $\Sigma_{n=0}^{\infty} (2^n+3^n)/4^n$ *Answer:* 6

(d) The first four terms (i.e., the constant term through the x^3 term) of the Maclaurin series for $(1+x+x^2)^{-1}$ *Answer:* $1-x+x^3$. (*You can also show a term* "$+0x^2$", *but you don't need to.*)

2. (36 points, 12 points each) For each of the items listed below, either *give an example* with the property stated, or give a brief reason why *no such example exists.* (If you give an example, you are *not* asked to show that it has the asserted property.)

(a) A power series whose interval of convergence is the open interval $(0, 100)$.

Answer: $\Sigma_{n=0}^{\infty} (x-50)^n/50^n$. (*For this and the other parts of this question, there are other examples you could have given than the ones shown here.*)

(b) A bounded sequence of real numbers $a_0, a_1, a_2, \ldots,$ which does not converge.

Answer: $a_n = (-1)^n$

(c) A power series which converges at $x=-1$ but at no other point. *Answer:* $\Sigma_{n=0}^{\infty} n!\,(x+1)^n$.

3. (16 points) Suppose $a_0, a_1, a_2, \ldots, a_n, \ldots$ are real numbers, and that r and s are nonzero real numbers with $|r| < |s|$. Prove that if the sequence $|a_0|, |a_1 s|, |a_2 s^2|, \ldots, |a_n s^n|, \ldots$ is bounded, then the series $\Sigma_{n=0}^{\infty} a_n r^n$ converges absolutely.

This was part of a lemma that I proved in class, in proving the theorem on radii of convergence of power series; so of course you cannot quote that lemma, or the theorem on radii of convergence, in proving this. But you can use any of the earlier results we had concerning sums of series.

Though remembering the lecture might help you do this problem, it is not necessary. The proof is a straightforward application of earlier results in the text.

Answer: The boundedness of the sequence of terms $|a_n s^n|$ *means that there exists a constant* $C \geq 0$ *such that* $|a_n s^n| < C$ *for all* n. *Now* $|a_n r^n| = |a_n s^n|\,|r/s|^n \leq C\,|r/s|^n$, *so the series* $\Sigma_{n=0}^{\infty} |a_n r^n|$ *has each term* $\leq$ *the corresponding term of the series* $\Sigma_{n=0}^{\infty} C\,|r/s|^n$. *The latter is a geometric series with ratio* $|r/s| < 1$, *and so converges.*

So $\Sigma_{n=0}^{\infty} |a_n r^n|$ *converges by the comparison test, which says that* $\Sigma_{n=0}^{\infty} a_n r^n$ *converges absolutely.*

assumes seq. odd : 7

Reminder: The reading for Monday, October 27, is #22