Math 113: Midterm 2
Prof. Beth Samuels
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Name :

Student ID Number:

Instructions: This is a closed-book test. Each problem is worth 20 points. Read the questions carefully, and show all your work. All work should be done on the exam paper. Additional white paper is available if needed. Good luck.

Problem	Score
1	
2	
3	
4	
5	
6	
7	
Total	

(1) Let G be the group of rotations of a cube. Consider G acting on the set of vertices, edges, and faces. Use the Counting Formula to compute the order of G in these three different ways.

(2) (a) Let G be a group. Define **the conjugacy classes** and the **class equation** of G.

(b) Determine the class equation for D_4.

(3) Let $|G| = p^3q$, where p and q are prime numbers. Show that either G has a normal Sylow p-subgroup, a normal Sylow q-subgroup, or $p = 2$ and $q = 3$.

(4) Classify groups of order 26. Prove your answer.

(5) (a) Define the **commutator** of two elements in a group, and define **the free abelian group** on $\{x, y\}$.

(b) Let G be a group. The commutator subgroup G' of G is the smallest subgroup containing all the commutators of G. Let N be a normal subgroup of G. Prove that G/N is abelian if and only if $G' \subset N$.

(6) (a) Define a **field** and an **ideal** of a ring.

(b) Let R be a ring which is not the zero-ring. Show that R is a field if and only if R has exactly two ideals.

(7) Let R be a ring, and let I and J be two ideals in R. Show that $I \cap J$ is an ideal in R.