

Math 1A Midterm 1 2005-9-29 11:00-12:30pm. R. Borcherds

You are allowed 1 sheet of notes. Calculators are not allowed. Each question is worth 3 marks, which will only be given for a clear and correct answer.

1. Find the domain of the function  $g(u) = \sqrt{u} + \sqrt{2-u}$ .
2. Sketch the graph of  $y = |x^2 - 2x|$ .
3. Find a formula for the inverse of the function  $f(x) = 1 + e^{x^3}$ .
4. Sketch the graph of a function  $f$  that satisfies the conditions

$$\lim_{x \rightarrow 0^-} f(x) = 1, \lim_{x \rightarrow 0^+} f(x) = -1, \lim_{x \rightarrow 1^-} f(x) = 1, \lim_{x \rightarrow 1^+} f(x) = -1, f(2) = 1.$$

5. Evaluate the limit

$$\lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 - 1}$$

6. How close to 2 do we have to take  $x$  so that  $5x + 3$  is within a distance of 0.01 from 13?
7. Find the numbers at which  $f$  is discontinuous, where  $f$  is defined by  $f(x) = x^2$  if  $x \leq 1$ ,  $f(x) = 1/x$  if  $1 < x < 3$ ,  $f(x) = 1/2 + \sqrt{x-3}$  if  $x \geq 3$ .

8. What is

$$\lim_{x \rightarrow -\infty} \frac{(3x+1)(4x+1)}{(x+1)(2x+1)}$$

9. A curve has equation  $y = f(x)$ . Write an expression for the slope of the secant line through the points  $(3, f(3))$  and  $(x, f(x))$ , and write an expression for the slope of the tangent line at  $(3, f(3))$ .
10. If  $g(x) = x^3 + x^2 + x + 1$ , find  $g'(0)$  and use it to find an equation of the tangent line to the curve  $y = x^3 + x^2 + x + 1$  at the point  $(0, 1)$ .
11. Sketch the graph of the function  $y = x^2|x|$ , say where it is differentiable, and find a formula for its derivative.
12. Differentiate the function  $y = e^{x+1} + 4\pi^2 + (x+1)/\sqrt{x}$
13. At what point on the curve  $y = 1 + 2e^x - 3x$  is the tangent line parallel to the line  $3x - y = 5$ ?
14. Differentiate  $xe^x(\sqrt{x} + 1)$
15. Differentiate

$$\frac{x^2}{1 + 1/x}$$