Math 53

Alan Weinstein, Fall 2005

First Midterm Exam, Tuesday, September 27th 2005

Instructions. BE SURE TO WRITE YOUR NAME AND YOUR GSI'S NAME ON YOUR BLUE BOOK. Read the problems very carefully to be sure that you understand the statements. All work should be shown in the blue book; writing should be legible and clear, and there should be enough work shown to justify your answers. **Indicate the final answers to problems by circling them.** [Point values of problems are in square brackets. The total point value is 45, for 15% of your course grade.]

PLEASE HAND IN YOUR SHEET OF NOTES ALONG WITH YOUR BLUE BOOK. YOU SHOULD ***NOT*** HAND IN THIS EXAM SHEET.

1. [12 points]
a. Sketch the curve with parametric equations $x = \cos t,\ y = 2\sin t$.
b. What kind of curve is this? (square, parabola,...etc.)
c. Find an equation of the form $ax + by + c = 0$ for the line which is tangent to this curve at the point where the parameter t has the value t_0.
d. For which values of t_0 does this tangent line cross the y-axis? (You may use the picture to check your answer, but not to justify it.)

2. [6 points] Every equation of the form $ax^2 + by^2 + cz^2 = d$, where a, b, c, and d are all nonzero real numbers, describes a hyperboloid of one sheet, a hyperboloid of two sheets, an ellipsoid, or "the empty surface". (A sphere may be considered as an ellipsoid with three equal axes.) Explain clearly how to tell, given the numbers a, b, c, and d, which kind of surface it describes.

3. [7 points]
a. Use the cross product to find numbers p, q, r, and s such that the plane $px + qy + rz + s = 0$ goes through the points $(1,0,0)$, $(0,2,0)$, and $(0,0,3)$.
b. Now find a DIFFERENT set of numbers p', q', r', and s' such that the plane $p'x + q'y + r'z + s' = 0$ still goes through the points $(1,0,0)$, $(0,2,0)$, and $(0,0,3)$.

4. [8 points]
Let

$$f(x,y) = \frac{x+y}{\sqrt{x^2+y^2}},\quad (x,y) \neq (0,0).$$

a. Write an equation in cylindrical coordinates for the graph of f.
b. Does the limit

$$\lim_{(x,y)\to(0,0)} f(x,y)$$

exist? Explain why or why not.
c. Does the graph contain any straight lines (perhaps with a single point missing)? Explain why or why not.

5. [12 points] Consider the curve described in polar coordinates by $r = 2 + \cos 2\theta$.
a. Explain, without doing any computation, why the area enclosed by the curve must be less than 9π.
b. Compute the area enclosed by the curve.
c. Explain why your result in part (b) implies that the curve cannot be contained within the rectangle with vertices at $(\pm 3, \pm 1)$.
d. Sketch the curve.