Math 32, Fall 2005
Instructor: Alex Diesl
December 14, 2005

Final Exam

Your Name: _____
SID: _____
Section: _____

Directions:

This is a 170 minute exam. Please do not start until instructed to, and please stop working when time is up. Read directions, work carefully and check your answers. Make sure that your final answer is circled (or in some other way identified). If you are running out of room, use the back of the page you are working on and so indicate. If you need scratch paper or have a question, please raise your hand. Be advised, however, that we may not be able to answer your question. We will, however, give you as much scratch paper as you want.

Some of the problems on the exam have multiple parts. You do not necessarily have to complete the parts in the order indicated. The parts ARE related, though.

1. *(5 points)* Pick any 5 angles θ (in radians) in the interval $[0, \frac{\pi}{2}]$. Fill in the table below with each angle, its cosine, and its sine.

	θ	$\cos(\theta)$	$\sin(\theta)$
1.			
2.			
3.			
4.			
5.			

2. (a) *(4 points)* The points A and B have polar coordinates $(-3, \frac{\pi}{4})$ and $(1, \frac{\pi}{6})$, respectively. Give the rectangular coordinates of A and B.

(b) *(2 points)* The point C has rectangular coordinates $(\sqrt{3}, -1)$. Write the polar coordinates for C in four distinct ways.

3. Let $f(x) = \log_2(\sqrt[3]{x+1} - 1)$.

   (a) *(4 points)* Find $f^{-1}(x)$.

   (b) *(4 points)* Solve for x: $f(x) = 1$

4. (a) *(4 points)* Find all solutions (real and complex) of $x^3 - 8 = 0$.

   (b) *(2 points)* Factor $x^3 - 8$ completely into linear factors.

5. *(8 points)* Graph the two parabolas $y = 2x^2 + 4x + 1$ and $y = -x^2 - 4x + 4$ on the same set of axes. Label the vertices, all intercepts and all points of intersection.

6. Evaluate:

   (a) *(2 points)* $\sin^{-1}(\frac{1}{2})$

   (b) *(2 points)* $\sin^{-1}(\sin(\frac{2\pi}{3}))$

   (c) *(2 points)* $\sin^{-1}(\sin(3))$

7. *(8 points)* Solve the following system for x, y and z:

$$\begin{aligned} x + y \quad -z &= \quad 2 \\ 2x + y \quad -3z &= -1 \\ -x + y \quad +3z &= \quad 5 \end{aligned}$$

8. *(8 points)* Let $f(x) = x^2$ and let $g(x) = x - 1$. Find all values of x for which

$$f(g(f(x)) = 3x^3 + x^2 - x + 1.$$

9. (a) *(4 points)* Factor $x^4 + 4$ into two quadratic polynomials.
(Hint: Write $x^4 + 4$ as $x^4 + 4x^2 + 4 - 4x^2$.)

(b) *(4 points)* Find all solutions to $x^8 - 16 = 0$.

10. *(7 points)* Solve for x and y:

$$\begin{aligned} 3\log_2(x) + \log_3(y) &= -5 \\ -\log_2(x) - \log_3(y) &= 1 \end{aligned}$$

11. *(7 points)* Solve for x, $0 \leq x < 2\pi$:

$$4\sin^4(x) - 7\sin^2(x) + 3 = 0$$

12. *(7 points)* Graph $y = x^2 - 2x - 3$ and $y = 1 - |x|$ on the same set of axes. Label the vertices, intercepts and any points of intersection.

13. Let $p(x) = x^6 - 2x^5 - 4x^4 + 6x^3 + 7x^2 - 4x - 4$.

    (a) *(2 points)* List all possible rational roots of $p(x)$.

    (b) *(6 points)* Factor $p(x)$ completely and find all roots of $p(x)$.

(c) *(4 points)* Solve $p(x) \geq 0$ and write the answer in interval notation.

(d) *(4 points)* Graph $y = x^6 - 2x^5 - 4x^4 + 6x^3 + 7x^2 - 4x - 4$ and label all intercepts.

14. *(7 points)* Graph $y = 2\sin(2x) - 1$ for $0 \leq x \leq 2\pi$. Label all intercepts.

15. *(6 points)* Determine the domain of

$$F(x) = \cot(x) + \sqrt{\frac{1}{x+1} - \frac{1}{x-1} - 2}.$$

Write the answer in interval notation.

16. *(5 points)*

(a) Solve for x: $7^x = 3(2^{3x-2})$

(b) *(2 points)* Is the solution positive or negative. Explain.

Extra Credit *(2 points)* Unscramble the letters given in capitals to find the two words that complete the sentence:

Precalculus is the marriage of GRAB ALE and ME GOT RYE.