Mathematics 1A, Fall Semester 2003
Instructor: Garth Dales
October 30, 2003

Midterm Examination 2

Your Name: ______________________________

Your SID: ______________________________

Your GSI's Name and Section Time: ______________________________

Directions:

- Do not open your exam until your are instructed to do so.
- You may not use any external aids during the exam: NO books, NO lecture notes, NO formula sheets, NO cell phones.
- Answers without explanation will not receive credit. *You must show and justify your work.* If necessary, use the backs of the pages or the extra pages attached to your exam, and indicate you have done so.
- When time is called, you must stop working and close your exam.
- All questions are worth 12 points each (for a total of 60 points).

Good Luck!

Score			
1.(a)	(b)		
2.(a)	(b)	(c)	(d)
3.(a)	(b)		
4.(a)	(b)	(c)	
5.			
Total	/60		

1. (a) *(4 points)* Let P_n be a statement that may or may not hold for $n = 1, 2, 3, \ldots$. State the *principle of mathematical induction*, which explains what we must show to prove P_n for each $n = 1, 2, 3, \ldots$.

   (b) *(8 points)* Let $f(x) = x^{1/2}$ for $x > 0$. Prove that

$$f^{(n)}(x) = (-1)^{n+1} \frac{(2n-2)!}{2^{2n-1}(n-1)!} \cdot \frac{1}{x^{(2n-1)/2}}$$

   for $x > 0$ and $n = 1, 2, 3, \ldots$.

2. Calculate the derivatives of each of the following functions. (Show your work, but there is no need to give the reason for each step.)

   (a) *(4 points)* $y = \log(x^4 \sin^2 x)$

   (b) *(4 points)* $y = (\sin x)^x$

   (c) *(2 points)* $y = \log(\log x)$

   (d) *(2 points)* $y = \sinh^2 x$

3. (a) *(8 points)* The altitude of a triangle is increasing at a rate of 1 cm/min while the area of the triangle is increasing at a rate of 2 cm^2/min. At what rate is the base of the triangle changing when the altitude is 10 cm and the area is 100 cm^2?

(b) *(4 points)* Use l'Hôpital's rule to calculate

$$\lim_{x \to 0} \frac{\sin x - e^x + 1}{x^2}$$

4. (a) *(4 points)* Find the (absolute) maximum and minimum values of the function

$$f(x) = x^3 - 6x^2 + 9x + 2$$

on the closed interval $[-1, 4]$.

(b) *(3 points)* Give a careful statement of Rolle's theorem (but do not give the proof).

(c) *(5 points)* Let f and g be continuous functions on the closed interval $[a,b]$ such that f and g are differentiable on the open interval (a,b). Suppose that $g'(x) \neq 0$ for each x with $a < x < b$. Deduce from Rolle's theorem that there exists c with $a < c < b$ such that

$$\frac{f(b)-f(a)}{g(b)-g(a)} = \frac{f'(c)}{g'(c)}.$$

5. *(12 points)* Sketch the graph of the function

$$f(x) = \frac{x^2}{x^2+3} \qquad (x \in \mathbb{R}).$$

In particular, find any intercepts, symmetry, any local maxima and minima, any points of inflection, and any asymptotes to the curve. Coordinate axes are provided for you on the next page.