

## 2ND MIDTERM EXAMINATION

Open book, open notes. In your proofs, you may use any result from the lectures, from the sections we've covered in the textbook, or from the homework exercises. Each question is worth 10 points.

1. Let  $X$  and  $Y$  be metric spaces. Let  $f : X \rightarrow Y$  and  $g : X \rightarrow Y$  be continuous functions. Prove that the set  $F = \{x \in X : f(x) = g(x)\}$  is closed.
2. Let  $A$  be a compact subset of  $\mathbb{R}$  and  $B$  a closed subset of  $\mathbb{R}$ . Prove the set

$$A + B = \{a + b : a \in A, b \in B\}$$

is closed.

3. Let  $X$  and  $Y$  be metric spaces. Let the sequence  $(f_n)_1^\infty$  of uniformly continuous functions from  $X$  into  $Y$  converge uniformly to the function  $f$ . Prove  $f$  is uniformly continuous.
4. Let  $c$  be the space of convergent sequences  $s = (s_n)_1^\infty$ , with the usual metric. Let the function  $f : c \rightarrow c$  be defined by  $f(s) = (s_n^2)_1^\infty$ . Prove  $f$  is continuous.
5. Let  $X$  be a metric space. Recall that the diameter of a subset  $A$  of  $X$  is defined by

$$\text{diam}(A) = \sup\{d(x, y) : x, y \in A\}.$$

Suppose  $K_1, K_2, \dots$  are nonempty compact subsets of  $X$  such that  $K_{n+1} \subset K_n$  for all  $n$ , and let  $K = \bigcap_{n=1}^\infty K_n$ . Prove that

$$\text{diam}(K) = \lim_{n \rightarrow \infty} \text{diam}(K_n).$$