

MATH 1B, Lecture 3
Sarason

March 14, 1996

MIDTERM EXAMINATION

Name (Printed) _____

Signature _____

TA _____

Section time _____

Closed book. No calculators.

SHOW YOUR WORK. Cross out anything you have written that you do not want the grader to consider.

The points for each problem are in parentheses.
Perfect score = 65.

| | |
|--------------|--|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| Total | |
| Grade points | |

1. (5) Evaluate the limit: $\lim_{n \rightarrow \infty} n^2 \ln\left(1 + \frac{1}{n^2}\right)$

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2. (15) Suppose $\sum_{n=1}^{\infty} a_n$ is an infinite series whose n-th partial sum is given by $s_n = 1 + \frac{(-1)^{n+1}}{n+1}$.

(a) What is a_n ? In particular, what are a_1, a_2, a_3 ?

(b) Does the series converge? If so, what is its sum? Explain.

(c) Does the series converge absolutely? Explain.

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3. (15) Determine whether the following series converge or diverge. Explain your reasoning. Be sure to make clear which convergence test or tests you are using.

(a) $\sum_{n=1}^{\infty} e^n (-e^n)$

(b) $\sum_{n=1}^{\infty} \ln\left(1 + \frac{1}{n^2}\right)$

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4. (15) Find the radii of convergence of the following power series.
(Do not check endpoints.) Justify your answers.

(a) $\sum_{n=0}^{\infty} 2^{4n}(x-3)^{3n}$

(b) $\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2} x^{3n}$

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5. (10) Write down the first three nonvanishing terms in the Maclaurin series of the function $f(x) = x^2(1-x^2)^{-1/3}$. Do the same for the functions f' and f'' .

6. (5) Let $\{c_n\}_{n=0}^{\infty}$ be a convergent sequence with a nonzero limit L . Find the radius of convergence of the power series $\sum_{n=0}^{\infty} c_n x^n$. Explain your reasoning.