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Math 128A. Midterm #2. 4 April 2003.

Choose 3 of 4 problems: $\square$ $\square$ $\square$

1. Assume a numerical approximation N to a quantity M satisfies

$$N = M + 3h + 5h^2 + O(h^3) \quad \text{as } h \to 0,$$

and we know three specific values

$$N(h) = N_1, \quad N(h/2) = N_2, \quad N(h/3) = N_3.$$

Construct an approximation N_{123} with

$$N_{123}(h) = M + O(h^3).$$

2. The quadrature formula

$$\int_0^1 f(x)\,dx = c_{-1} f(-1) + c_0 f(0) + c_1 f(1)$$

Note!

has degree of precision (accuracy) 2. Find c_{-1}, c_0 and c_1. For what p is the error given by

$$K f^{(p)}(\xi) \, ?$$

3. Suppose c_{-1}, c_0 and c_1 are numbers such that the quadrature formula

$$\int_0^1 f(x)\,dx = c_{-1} f(-1) + c_0 f(0) + c_1 f(1).$$

has precision 2.

a) Find weights d_0, d_1, d_2 such that the quadrature formula

$$\int_0^1 f(x)\,dx = d_0 f(0) + d_1 f(1) + d_2 f(2)$$

has precision 2.

b) Find a compound rule, involving the c's and d's such that

$$\int_0^1 f(x)\,dx = \sum_{j=0}^{n} w_j f(jh) \quad \left(h = \frac{1}{n}\right) + O(h^3) \quad \text{as } h \to 0.$$

Express the weights w_j in terms of c's and d's.

4. (a) Show that $y' = y$, $y(0) = 1$ has a unique solution $y(t)$ on the interval $[0,1]$.

(b) Derive a Taylor method of order two for the initial value problem in (a).