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Spring 2001, Math 114

First Midterm

23 February, 2001
12:10-1:00 PM

1. (30 points, 10 points each.) Complete each of the following definitions. (Do not give examples or other additional facts about the concepts defined.)

(a) If $L:K$ is a field extension, and α is an element of L, then α is said to be *algebraic* over K if

(b) If K is a field and X a subset of K, then the subfield of K generated by X is defined to be

(c) If K is a subfield of each of the fields M and L, then a monomorphism $\varphi: M \to L$ is said to be a *K-monomorphism* if

2. (50 points; 10 points each.) For each of the items listed below, either *give an example,* or give a brief reason why *no example exists.* (If you give an example, you do *not* have to prove that it has the property stated.)

(a) A field extension $L:K$, and a polynomial $f \in K[t]$ which is irreducible over K but reducible over L.

(b) A field extension $L:K$, and a polynomial $f \in K[t]$ which is reducible over K but irreducible over L.

(c) A finite field extension $L:K$ such that, regarding K as an intermediate field, we have $K^{*\dagger} = K$.

(d) A finite field extension $L:K$ such that, regarding K as an intermediate field, we have $K^{*\dagger} \neq K$.

(e) A finite field extension $L:K$ and three distinct automorphisms $\alpha, \beta, \gamma \in \Gamma(L:K)$ such that for all $x \in L$, $\alpha(x) + \beta(x) + \gamma(x) = 0$.

3. (20 points) **(a)** (10 points) Suppose $L:K$ is a field extension, and H a subgroup of $\Gamma(L:K)$. Recall that $H^\dagger$ means $\{x \in L \mid (\forall\ \alpha \in H)\ \alpha(x) = x\}$. In Lemma 7.2 Stewart shows that $H^\dagger$ is a subfield of L containing K. Prove the two parts of that statement saying that $H^\dagger$ is closed in L under addition and under multiplication.

Since I am asking you for the proof of a result in Stewart, you may not, of course, cite that result, or any result proved after it, in your proof.

(b) (10 points) Prove that $H^\dagger$ is also closed under multiplicative inverses (i.e., that if $x \in H^\dagger$ and $x \neq 0$ then $x^{-1} \in H^\dagger$). Stewart is not explicit about this point. You should supply the details, though he omits them. (Suggestion: Use the equation by which x^{-1} is defined. You may assume without argument that every automorphism of a field sends 1 to 1, but no other facts about automorphisms other than the definition.)