Department of Mathematics, University of California, Berkeley

Math 1A

Alan Weinstein, Fall 2000

First Midterm Exam, Thursday, September 28, 2000

Instructions. Be sure to write on the front cover of your blue book: (1) your name, (2) your Student ID Number, (3) your TA's name (Eric Antokoletz, Victor Deletang, Matthieu Hamel, Andre Henriques, Di-An Jan, Chu-Wee Lim, Russell O'Connor, Alf Onshuus, Emmanuel Py, Shahed Sharif, Dan Stevens, Karla Westphal, or Alexander Woo).

Read the problems very carefully to be sure that you understand the statements. Show all your work as clearly as possible, and circle each final answer to each problem. Remember: if we can't read it, we can't grade it.

1 . [12 points] Let $f(x) = x/(x+1)$ and $g(x) = -1/(x+1)$.
(A) Find $f'(x)$.
(B) Find $g'(x)$.
(C) Find the equation of the tangent line to the graph $y = x/(x+1)$ which goes through the point $(1, 1/2)$.
(D) Find the horizontal and vertical asymptotes of the graph of f.
(E) Find four different functions such that the derivative of each function is equal to $3x^2$.
(F) Look at your answers in parts (A), (B), and (E); what do they suggest about the relation between the functions f and g?

2 . [6 points] Find the following limit. [Hint: write $\frac{d}{dx}e^x$ as a limit.]

$$\lim_{x \to 1} \frac{e^x - e}{x^2 - 1}.$$

3 . [8 points] If $f(2) = 5$ and $f'(2) = 3$, find each of the following, or say if there is insufficient information to find it.
(A) $(1/f)'(2)$. (B) $\frac{d}{dx}(\frac{1}{3}f(x)^2 + 17)$ at $x = 2$. (C) $f(3)$. (D) $f(2.001)$.

4 . [6 points] A round solar panel is to be constructed with radius 10 meters, but errors in construction are inevitable. **Using the properties of inequalities to justify your answer**, find a number δ with the property that, if the radius of the collector is within δ meters of the specified size, the area of the collector will be within 2 square meters of the desired area of 100π square meters (Assume that, even with errors, the collector is still perfectly round.)

5 . [13 points] Let Q be the function defined by the equations $Q(t) = 0$ for $t \leq 0$ and $Q(t) = 16t^2$ for $t \geq 0$.
(A) Sketch the graph of Q.
(B) Find $Q'(t)$ for all those t where Q is differentiable.
(C) Sketch a graph of Q'.
(D) Where is Q continuous? **Why?**
(E) Where is Q' differentiable?
(F) Find a function R such that $R'(t) = Q(t)$ for all t, and sketch its graph.